

Using CLAHE Method in the Classification of Fatty Mammograms with Abnormalities

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ABSTRACT

Objective: To classify fatty mammograms using the Contrast Limited Adaptive Histogram Equalization (CLAHE) method, integrating additional techniques to improve classification accuracy. Methods: The proposed approach combines wavelet-based denoising, morphological operations, and second-order statistical moment analysis. Experimental validation was carried out using the Mammographic Image Analysis Society Digital Mammography Database (MIAS-DMD), which includes 106 mammographic images (66 normal and 40 with pathological findings). Results: The proposed method achieved a sensitivity of 100% and a specificity of 96.97% in the classification of fatty mammograms. Conclusion: The results demonstrate the effectiveness of the method in accurately classifying fatty mammograms. Achieving 100% sensitivity means that all abnormal cases were correctly identified, while a specificity of 96.97% indicates a very low false-positive rate. This balance suggests that the method is both reliable and robust, making it a valuable tool to support clinical decision-making in breast cancer screening.

Mathematics Subject Classification: Primary: 68U10; Secondary: 68U07.

Keywords: Mammography, Contrast Limited Adaptive Histogram Equalization, Wavelet Analysis, Morphological Operations.

KEY MESSAGES

- The proposed method, which combines CLAHE, wavelet-based denoising, morphological operations, and second-order moment analysis, achieved excellent classification performance on fatty mammograms.
- Experimental results on the MIAS-DMD dataset show a sensitivity of 100% and a specificity of 96.97%, indicating both high accuracy and low false positive rates.
- This balance between sensitivity and specificity highlights the method's reliability and potential for clinical application in breast cancer screening.

1. INTRODUCTION

Breast cancer remains one of the leading causes of mortality among women worldwide. Mammography is a widely adopted technique for the early detection of this disease. However, approximately 10% of cancerous lesions may go undetected by radiologists due to the inherently low contrast of mammographic images. Consequently, enhancing image contrast is critical to improving the diagnostic performance of mammography.

Numerous contrast enhancement techniques have been proposed in the literature [1, 2, 3]. Among them, the Contrast Limited Adaptive Histogram Equalization (CLAHE) method has been extensively applied across various domains. For instance, [4] introduces a face recognition approach robust to facial expressions, occlusions, and particularly to illumination variations; [5] proposes an algorithm for enhancing visibility in surveillance videos affected by fog and/or rain; and [6] presents an efficient technique for infrared image enhancement within general infrared imaging systems.

In the domain of mammography, several computer-aided diagnosis (CAD) systems have been developed to support lesion detection. These include methods based on convolutional neural networks (CNNs) with multiple circular paths for mass detection [7], hybrid approaches combining artificial neural networks (ANNs) with wavelet-based subband decomposition for microcalcification detection [8], CAD systems utilizing ultrasonographic features and ANN classifiers for breast nodule malignancy classification [9], and algorithms employing two-dimensional linear phase nonseparable wavelet transforms combined with binary tree classifiers for detecting spiculated lesions [10].

The objective of this work is to provide radiologists with a reliable second-opinion tool to assist in the analysis of digital mammograms and improve the accuracy of breast cancer detection. The proposed method integrates wavelet analysis, morphological operations, and the CLAHE technique within a unified framework for image enhancement and classification.

To characterize tissue texture, several statistical features were extracted from the processed images, including entropy, standard deviation, mean, contrast, and energy.

The proposed algorithm was evaluated using the Mini-Mammographic Image Analysis Society Database (MIAS) [11]. In this database, all images have been cropped and zero-padded to a resolution of 1024×1024 pixels, with a spatial resolution of 200 microns and an 8-bit grayscale range (0–255).

MIAS comprises a total of 322 mammograms (right and left breast) from 161 patients, of which 206 were classified as normal. The dataset includes images with various tissue types: fatty (106), fatty-glandular (104), and dense-glandular (112). Detected abnormalities are categorized as calcifications, well-defined circumscribed masses,

spiculated masses, architectural distortions, asymmetries, and other ill-defined masses.

Additionally, the database provides a detailed annotation file listing image metadata, including the type of abnormality, the (x, y) coordinates of the abnormality center, and an approximate radius (in pixels) of a circle enclosing the lesion.

This paper is organized as follows: Section 1 provides the introduction. Section 2 describes the definitions and preliminary concepts used throughout the work. Section 3 outlines the proposed methodology. Experimental results are presented and discussed in Section 4. Finally, Section 5 contains the conclusions.

2. PRELIMINARIES AND DEFINITIONS

Image texture is defined as a function of the spatial variation in pixel intensities (gray levels). Images of real-world objects rarely contain regions of completely uniform intensity [12]. The most commonly used statistical descriptors for quantifying the texture of an image I of size $M \times N$ are the mean pixel intensity (μ), which reflects the general brightness or darkness of the region, and the standard deviation (σ), which provides insight into the local contrast.

Texture analysis is a widely used technique in medical imaging, as it facilitates the differentiation of lesion patterns based on tissue density in mammograms. Several studies, such as [13, 14], have explored its application in this context. Furthermore, texture descriptors can be utilized to assess the density of abnormal masses [15].

Various approaches to texture-based feature extraction in mammographic images have been reported, differing mainly in the scale and granularity of the analyzed regions. For instance, [16] employs gray-level co-occurrence matrices (GLCMs) to compute statistical texture measures, while [17] adopts multiresolution techniques such as the ridgelet transform for image tomography. In this study, we utilize several texture features derived from the GLCM.

The gray-level co-occurrence matrix (GLCM) is a square matrix that quantifies the frequency of co-occurrence of pixel intensity pairs at a specified spatial relationship within the image. Formally, for an image I of size $M \times N$, a GLCM is defined for a given offset $(\Delta x, \Delta y)$ which corresponds to a distance d and direction θ and records the number of times a pixel with gray level i is spatially adjacent (in the specified direction and distance) to a pixel with gray level j .

To construct the GLCM, we define a position operator P and a preliminary matrix A , where each entry $a_{i,j}$ denotes the number of occurrences of gray-level pair (i, j) at the specified offset. The number of gray levels determines the order of matrix A . The normalized GLCM is then obtained by dividing each element of A by the total sum of all its entries, yielding a probability

distribution.

To reduce computational complexity while maintaining discriminative power, each image is quantized into 8 gray-level bands. For the construction of the GLCM, we adopt a direction $\theta = 0^\circ$ and a distance $d = 1$. From the resulting GLCM, a set of textural features such as contrast, homogeneity, correlation, and energy can be extracted to characterize image regions [18].

Image segmentation is the process of partitioning an image into mutually exclusive and spatially coherent regions based on specific attributes, such as gray-level intensity or texture. This technique enables the isolation of distinct objects or anatomical structures from the background, thereby facilitating further analysis.

In the context of medical imaging, and particularly in mammography, segmentation plays a critical role in identifying regions that may correspond to lesions or abnormalities. Accurate segmentation is essential for the subsequent stages of feature extraction and classification. To effectively segment mammographic images and localize potential pathological areas, it is necessary to introduce and understand the following key concepts.

Definition 2.1. Entropy (*Ent*) of a digital image I is defined as [19]:

$$Ent = - \sum_{i,j} p(i,j) \log_2(p(i,j))$$

where $p(i,j)$ is the relative occurrence frequency of two pixel values, one with intensity i and the other with intensity j .

Remark 2.2. Entropy (*Ent*) is a statistical measure of randomness that quantifies the degree of disorder or unpredictability in the distribution of pixel intensities within an image. It serves as an indicator of texture complexity, with higher entropy values corresponding to more heterogeneous or complex textures.

From an information-theoretic perspective, entropy reflects the amount of information or uncertainty associated with the gray-level distribution. An image with a highly uniform or ordered distribution of pixel intensities will exhibit low entropy, whereas an image with a more diverse or irregular distribution will result in higher entropy values.

Definition 2.3. Contrast (*Con*) of a texture is defined by:

$$Con = \sum_{i,j} (i - j)^2 GLCM(i,j)$$

Remark 2.4. Contrast (*Con*) is a statistical measure that quantifies the intensity variation between a pixel and its neighboring pixels across the entire image. It provides insight into the presence of abrupt changes in gray levels, which are typically associated with edges, texture boundaries, or fine details.

Mathematically, contrast evaluates the dispersion of the gray-level co-occurrence matrix (GLCM) elements relative to its main diagonal. A smooth or homogeneous texture yields low contrast values, indicating minimal intensity variation, whereas rough or irregular textures result in higher contrast values. For a perfectly uniform (constant) image, the contrast is equal to zero.

Definition 2.5. *Energy* (E) is defined as

$$E = \sum_{i,j} GLCM(i,j)^2$$

Remark 2.6. E is the sum of squared elements in the *GLCM*. Its range is [0 1] and it is 1 for a constant image. Energy is also known as uniformity or angular second moment.

The energy is related to the homogeneity of the distribution of the pixels of an image. Higher values of energy indicate more homogeneous images. The energy property gives an idea of the softness of texture.

2.1. Contrast Limited Adaptive Histogram Equalization.

Let I a grayscale image made of $M \times N$ pixels with intensity levels in the range $[0, L-1]$. The normalized histogram of I is given by

$$p_r(r_k) = \frac{n_k}{MN}, k = 0, 1, \dots, L-1$$

where r_k is the k -th intensity value and n_k is the number of pixels in I with intensity r_k . Note that $p_r(r_k)$ is an estimated of the probability of occurrence of intensity level r_k in an image.

The Contrast Limited Adaptive Histogram Equalization (CLAHE) algorithm was originally developed for medical imaging, in particular, for the enhancement of low-contrast images. Originally it was used the histogram equalization, a method in image processing of contrast adjustment based on the assumption that in a good contrast grayscale image, pixels values are uniformly distributed. It is obtained by the equation:

$$s_k = (L-1) \sum_{j=0}^k p_r(r_j), k = 0, 1, \dots, L-1$$

where s_k is the new distribution of the histogram.

Then it was used the Adaptive Histogram Equalization; this technique divides the image into a limited number of rectangular regions, typically of 64×64 pixels sized. In this case histogram

equalization is computed independently with L bins and it is used a bilinear interpolation. Since in nearly flat areas perhaps all pixels will be remaped to the maximum output value, it appears the problem of very noisy adapted image. This issue is solved by limiting the contrast (CLAHE) proposed in [20] and summarized in [21], by a clip limit for clipping histograms. Next, each histogram is redistributed in such a way that its height does not go beyond the clip limit. The clip limit β is obtained by:

$$\beta = \frac{MN}{L} \left(1 + \frac{\alpha}{100} (s_{max} - 1) \right)$$

where α is a clip factor. After thresholding, the pixels in excess in each bin are equally redistributed until they do not exceed β .

3. METHODOLOGY

We describe below the process followed in this work.

- (1) Take the mammogram images from the available database.
- (2) Improve the contrast of the grayscale image.
- (3) Eliminate the noise.
- (4) Apply the analysis wavelet decomposition.
- (5) Binarize.
- (6) Remove small objects.
- (7) Fill regions and holes.
- (8) Place the obtained image on top of the initial image.

1. The image is read.

An intensity image is a data matrix I , where each element represents the gray-level intensity of a corresponding pixel within a defined dynamic range. In other words, each entry in I encodes the luminance information of a single pixel in the image.

The intensity images used in this study are obtained from the MIAS database and are stored in PNG (Portable Network Graphics) format. After loading the image, a manual cropping step is performed to isolate and emphasize the region of interest (ROI), thereby facilitating subsequent processing and analysis.

2. Improve the contrast of the grayscale image.

The contrast of the grayscale image is enhanced by applying the Contrast Limited Adaptive

Histogram Equalization (CLAHE) technique. Unlike global histogram equalization methods, CLAHE operates locally on small, non-overlapping regions of the image, known as tiles, enabling adaptive contrast enhancement tailored to local image content.

The main parameters governing the CLAHE algorithm are as follows:

- Distribution (D). The contrast of each tile is enhanced such that the histogram of the output region approximates the desired histogram shape defined by the specified parameter. This parameter controls the target distribution for the pixel intensities within each tile. To ensure seamless transition between adjacent tiles and prevent artificial boundary artifacts, bilinear interpolation is applied to blend the neighboring regions smoothly. Moreover, the contrast enhancement in relatively homogeneous areas is limited to prevent excessive amplification of noise, thereby preserving the overall image quality and diagnostic relevance.
- Clip Factor (α). It is a real scalar in the range [0 100] that specifies a contrast enhancement limit. Higher numbers result in more contrast. It is established $\alpha = 0.8$ for all images.
- Number of Bins (NB). A positive integer scalar specifying the number of bins used to construct the histogram for the contrast enhancement transformation. Increasing the number of bins results in a finer dynamic range representation; however, this comes at the expense of increased computational complexity and reduced processing speed. In this study, we set the number of bins to $NB = 160$.
- Range (R). This parameter defines the range of intensity values in the output image. In the present study, the output range is constrained to match that of the original image I.
- Tiles (T). A two-element vector of positive integers specifying the number of tiles along the rows and columns, respectively. In this work, we use the values [7, 8].

The contrast of the image is enhanced by mapping the pixel intensity values of the input image to new values in the output image, such that a specified percentage of data is saturated at the low and high intensity ends of the input range. This saturation level is controlled by the parameter b . Figure 1 illustrates the effect of varying the parameter b on the display of the same mammographic image.

The contrast enhancement procedure is performed as follows: First, the entropy of each image is calculated, yielding values distributed within the interval [6.1, 7.4]. Mammograms with similar entropy values are grouped together by partitioning the initial interval into subintervals of length 0.1. Subsequently, additional statistical features such as standard deviation, mean, correlation, and energy are analyzed. Based on the similarities or differences observed in these characteristics, the parameter b is selected accordingly.

In general, the parameter b is chosen to be 0.98; however, to achieve optimal luminance, the value of b is adjusted as follows:

◊ $b = 1$ for all images with $\text{Ent}=6.1$ and furthermore if $\{ \text{Ent}=6.6, \mu \in [90 \ 105] \text{ and } \sigma \in [54 \ 60.9] \}$, $\{ \text{Ent}=6.7 \text{ and } \mu \in [83 \ 99] \}$, $\{ \text{Ent}=6.8 \text{ and } \mu \in [96 \ 97] \}$, $\{ \text{Ent}=6.9 \text{ and } \mu < 112 \}$, and $\{ \text{Ent}=7.1, \mu \in [96 \ 107] \text{ and } \sigma > 64 \}$.

◊ $b = 0.97$ for all images with $\text{Ent} = 6.2$ and also if $\{ \text{Ent}=6.8, \mu \in [104 \ 105] \text{ and } \sigma < 105 \}$, and $\{ \text{Ent}=7.1, \mu \in [93 \ 96] \text{ and } \sigma < 96 \}$.

◊ $b = 0.92$ if $\{ \text{Ent}=6.4, \mu \in [81 \ 82] \}$, $\{ \text{Ent}=6.6, \mu \in [93 \ 110] \text{ and } \sigma \in [64 \ 67] \}$, $\{ \text{Ent}=6.7, \mu \in [104 \ 116], \sigma \in [65 \ 70], \text{Con} \in [0.04 \ 0.08] \}$ and $\{ \text{Ent}=6.9 \text{ and } \mu \in [112 \ 113] \}$.

◊ $b = 0.9$ if $\{ \text{Ent}=7.0, \mu \in [87.3, 111.6] \text{ and } \sigma \in [65.4, 68.7] \}$ and $\{ \text{Ent}=7.2, \mu \in [84.2, 126.1], \sigma \in [58.4, 73.9] \text{ and } E \in [0.17, 0.23] \}$.

3. Eliminate the noise.

Various noise elimination methods have been proposed in the literature (see, for example, [22]). In this study, noise removal is performed in two stages. First, wavelet denoising is applied using the Daubechies 10 (db10) wavelet at level $n = 1$, employing global thresholding with a soft thresholding scheme [23]. Subsequently, wavelet packet denoising is performed, also using db10 at level $n = 1$, but with global thresholding combined with hard thresholding.

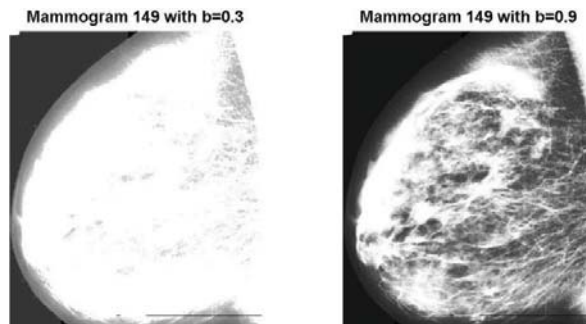


Figure 1. A mammogram image using different values of b .

Wavelet denoising aims to suppress noise while preserving important image features. This technique critically depends on the selection of a threshold T . We utilize the universal threshold proposed by Donoho [24], defined as

$$T = \sigma \sqrt{2 \log(mn)}$$

where σ is the estimated noise standard deviation and $m \times n$ is the total number of pixels in the image.

Thresholding can be classified as either *hard* or *soft*, both defined with respect to the

threshold T as follows:

- Hard thresholding: The signal value is retained if its magnitude exceeds the threshold, i.e.,

$$x_{hard} = \begin{cases} x, & |x| > T \\ 0 & |x| \leq T \end{cases}$$

- Soft thresholding: The signal is shrunk towards zero by the threshold value, given by

$$x_{soft} = \begin{cases} \text{sign}(x)(|x| - T), & |x| > T \\ 0 & |x| \leq T \end{cases}$$

The two-dimensional denoising process follows the same principles as the one-dimensional case, but employs two-dimensional wavelet transforms.

In this work, we adopt soft thresholding during wavelet denoising, which shrinks coefficients exceeding the threshold in magnitude. The continuity of the soft thresholding function offers several advantages, including improved mathematical tractability of denoising algorithms [23]. Moreover, soft thresholding possesses favorable theoretical properties that support its effectiveness.

Regarding wavelet packet denoising, the approach is fundamentally similar to wavelet denoising. The primary distinction lies in the increased flexibility of wavelet packet analysis, where both detail and approximation coefficients are decomposed further, allowing for a more refined and adaptive noise reduction process.

4. Apply the analysis wavelet decomposition.

Wavelet decomposition is performed using the Daubechies 10 (*db10*) wavelet at level $n = 1$, and the approximation coefficients at this level are reconstructed to obtain an intensity image. This reconstructed image serves as input for subsequent processing stages.

The wavelet decomposition of an image is carried out as follows: at each decomposition level, the image is divided into four subbands. Specifically, the two-dimensional wavelet transform decomposes the approximation coefficients at level j into four components: the approximation coefficients at level $j + 1$, and detail coefficients corresponding to three orientation: horizontal, vertical, and diagonal.

5. Binarize.

The grayscale image I is converted to a binary image based on a thresholding process. The resulting binary image BW assigns a value of 1 (white) to all pixels in the input image whose luminance exceeds the specified threshold level, while all other pixels

are assigned a value of 0 (black). In this study, we apply a global threshold to each mammogram using Otsu's method.

6. Remove small objects.

Finally, all connected components (objects) in the binary image containing fewer than P pixels are removed, resulting in a refined binary image. In this study, objects with fewer than 2050 pixels are eliminated.

7. Fill regions and holes.

Holes in the binary image are filled to improve the segmentation results. It is important to note that dilation is not employed, as the objective is to fill specific regions without expanding the entire set.

8. Place the obtained image on top of the initial image.

Finally, the processed image is overlaid on the original mammogram. The complete procedure is demonstrated using the mammographic image shown in Figure 2.

4. EVALUATION OF THE CLASSIFICATION METHOD AND RESULTS

Mammogram 274 with abnormality The algorithm indicates the abnormality

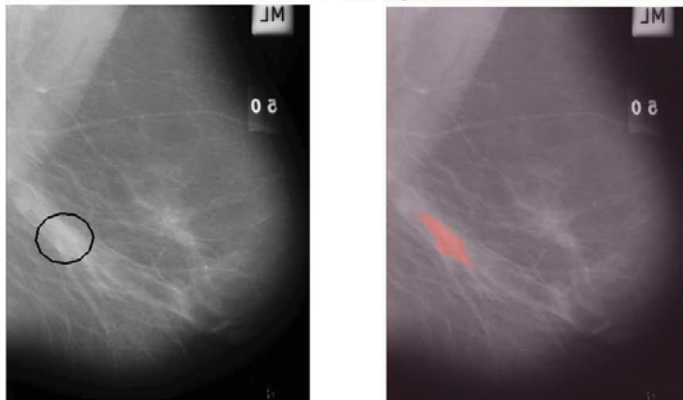


Figure 2. The algorithm indicates the abnormality in the image mdb274.

In general, sensitivity and specificity are the most commonly used metrics to evaluate the performance of a diagnostic test.

Sensitivity represents the proportion of true positive cases correctly identified by the test (i.e., the presence of an abnormality) and is defined as:

$$S = \frac{TP}{(TP + FN)}$$

Specificity measures the proportion of true negative cases that are correctly identified as such, and is defined by:

$$Es = \frac{TN}{(TN + FP)}$$

where FN, FP, TN and TP denote false-negative, false-positive, true-negative and true-positive diagnosis, respectively.

In tests on fatty mammograms from the MIAS database we obtain:

$$S(\%) = \frac{40}{40 + 0} 100 = 100\% \quad Es(\%) = \frac{64}{64 + 2} 100 = 96.97\%$$

Table 1 summarizes the classification results for the 106 mammographic images.

5. CONCLUSIONS

This paper presents preliminary results for the classification of fatty mammograms into two categories: normal and those containing lesions. Abnormal mammograms include circumscribed masses, architectural distortions, asymmetries, and other ill-defined masses. All fatty mammograms from the MIAS database were analyzed, and the proposed classification method achieved a sensitivity of 100% for detecting abnormal mammograms.

Table 1. 66 normal fatty mammograms using the algorithm we obtain 2 False Positive (FP).

mdb	TN	FP	mdb	TN	FP	mdb	TP	FN
mdb006	•		mdb237	•		mdb005	•	
mdb009	•		mdb246	•		mdb010	•	
mdb011	•		mdb247	•		mdb012	•	
mdb026	•		mdb251	•		mdb025	•	
mdb027	•		mdb255	•		mdb028	•	
mdb060	•		mdb268	•		mdb059	•	
mdb070	•		mdb272	•		mdb069	•	
mdb076	•		mdb273	•		mdb075	•	
mdb077	•		mdb293	•		mdb080	•	
mdb078	•		mdb294	•		mdb091	•	
mdb079	•		mdb297	•		mdb092	•	
mdb087	•		mdb298	•		mdb095	•	
mdb088	•		mdb299	•		mdb097	•	
mdb096	•		mdb300	•		mdb132	•	
mdb098	•		mdb301	•		mdb134	•	
mdb131	•		mdb302	•		mdb141	•	
mdb133	•		mdb303	•		mdb142	•	
mdb135	•		mdb304	•		mdb144	•	
mdb136	•		mdb305	•		mdb148	•	
mdb139	•		mdb306	•		mdb150	•	
mdb140	•		mdb307	•		mdb152	•	
mdb143	•		mdb308	•		mdb155	•	
mdb147	•		mdb309	•		mdb158	•	
mdb149	•	•	mdb310	•		mdb160	•	
mdb151	•	•	mdb311	•		mdb167	•	
mdb153	•		mdb313			mdb184	•	
mdb154	•					mdb195	•	
mdb156	•					mdb204	•	
mdb157	•					mdb206	•	
mdb159	•					mdb231	•	
mdb168	•					mdb238	•	
mdb173	•					mdb245	•	
mdb174	•					mdb248	•	
mdb183	•					mdb252	•	
mdb196	•					mdb256	•	
mdb203	•					mdb267	•	
mdb205	•					mdb271	•	
mdb229	•					mdb274	•	
mdb230						mdb312	•	
mdb232						mdb314		
Total	38	2		26	0		40	0

Ethics Statement

None. The study used only publicly available anonymized data from the MIAS database, and no human or animal subjects were directly involved.

Author Contributions Statement

L.A.-A. designed the study and performed the image preprocessing. M.B.-G. implemented the CLAHE method and carried out the classification experiments. D.L.-R. analyzed the results and contributed to the interpretation of data. L.A.-A. and M.B.-G. wrote the main manuscript

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