

An Approach to Impose Environmental Tax on the Mitigation of Greenhouse Gas Emission

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ABSTRACT

In this paper, an idea to impose environmental tax on the basis of greenhouse gas emission has been developed. A linear non homogeneous equation has been used to describe the dynamics of concentration of greenhouse gases which involves a term for environmental tax. A formula has been obtained to calculate amount of environmental tax to reduce the concentration of greenhouse gases to some extent in prescribed time. For fair imposition of environmental tax a compartmental model has been constructed here. Some numerical simulation also has been done to describe the results of the analysis.

Keywords: Greenhouse Gases, Industrialization, Environmental Tax, Compartmental Model, Tax Distribution.

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1 Introduction

Global warming is affecting our ecosystem very badly. Main driving force behind greenhouse effect is the emission of greenhouse gases CO_2 , N_2O and CH_4 (Wikipedia, 2019). Fossil fuel production and combustion, land use, cement and other industries, transportation, power plants and disposal of chlorofluorocarbons are the major sources of GHGs emission. There also exists some natural sinks of the greenhouse gases like large water bodies, forested area and dense vegetative area but their presence is not sufficient to compensate the net GHGs which are emitted in large amount by their emitters (UCSUSA, 2019). That's why world is facing a terrible problem named as global warming and standing on a critical stage (Wikipedia, 2019, Villabolo, 2011) because due to this temperature of the earth is increasing (Hansen, 2010) at an alarming rate which is resulting an increasing list of environmental problems such as ecological imbalance (Coon, 2001, Karl, Melillo and Peterson, 2009). The major consequences of

ecological imbalance are a decreased species diversity, due to habitat loss which corresponds to a reduced “species carrying capacity”, acid rain, global warming, depletion of ozone layer and natural calamities like flood, hurricanes, and cyclones nearby the oceans etc.. Smaller habitats can only accommodate smaller population, this result in an impoverished gene pool. Medical Journal “The Lancet” has recently published a report named as “Countdown on health and Climate Change” (Watts et. al. , 2017) which clearly reveals that the percentage of natural disasters was 90% of all disasters worldwide in last two decades and 46% increment is recorded in number of disasters due to poor health of ecosystem from 2000.

Combined result of different studies (Sharma and Dikshit , 2016, Ganga, Gurjar and Kumari , 2011) of the emission of greenhouse gases (CO_2 , N_2O , CH_4) in India's one of the most polluted city and its capital Delhi given by figures 1, 2, 3 and 4. Figure:1 reveals that contribution of domestic, waste, power plants, transport, agriculture and industries in atmospheric concentration of GHGs in year 1990, 2000 and 2014. Further, it demonstrate that except domestic sector contributions of all sectors are increasing drastically and also this increment is accelerating which is dangerous to our ecosystem. Waste materials, power plants, transport and industries are the main drivers of greenhouse emission. Figure 2, 3 and 4 are the pie charts showing the percentage of contribution of different sectors in year 1990, 2000 and 2014. These figures are revealing that contribution of industries and waste materials become more significant nowadays. In 1990 and 2000 power plants, transport and domestic sector were the major emitters of GHGs whereas in 2014 the role of domestic sector is replaced by waste materials.

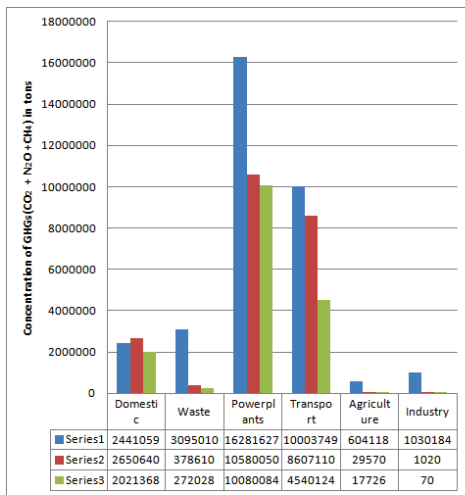


Figure:1

Series1,2 and 3 representing the data set of the years 2014, 2000 and 1990 respectively in Figure:1.

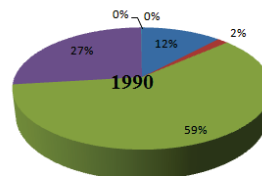


Figure:2

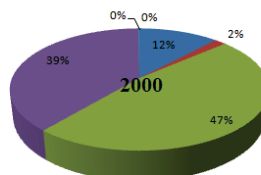


Figure:3

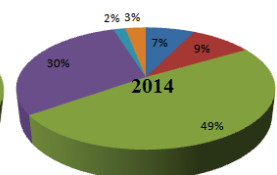


Figure:4

Domestic Waste Powerplants Transport Agriculture Industry

Further, it is also estimated that cities are 75% – 80% responsible for GHGs emission (Satterthwaite , 2008). All of the studies are revealing that most of the causes of global warming are man made. Policy makers, industrialists, mankind are totally ignoring the quality of ecosystem while making policies related to industrialization and population growth. There is no limitation which resist them on doing so and compel them for taking care of ecosystem.

Although industrialization is necessary for the development of the world but now its negative effects are more harmful than its benefits because nothing is more important than our nature and

human lives. It is a well known fact that by using low carbon fuels, solar energy and stopping burning MSW (Municipal solid waste) significant reduction can be achieved in the emission of these harmful gases but most of the people are not taking it seriously. At this time, we are at the critical stage (Wikipedia , 2019, Villabolo , 2011), so we need an additional option due to which a reduction in the emission of GHGs can occur. Making an oral observation on the emitters of GHGs, we can trivially conclude that the persons who are more economically strong are emitting GHGs in greater amount (Lin et. al. , 2011) because they are running industries, using vehicles, air conditioner unessentially, refrigerates, energy, poly items e.t.c. which are very much responsible for greenhouse gas emission.

Thus here seems a necessity of imposing some restriction in terms of tax on the people to prevent them from the indiscriminate exploitation of GHGs emitters. We name this tax as environmental tax.

2 Materials and Methods

Although, we know imposition of environmental tax is necessary to prevent excessive emission of GHGs but here is a question, how to impose it and on whom?

We can not impose environmental tax uniformly on everyone because that will be injustice with the people who are emitting GHGs in significant amount generally for the people from ruler areas, since urban areas are the major producer of GHGs (Satterthwaite , 2008). Future in urban area, each and every person is also not responsible for this huge amount of emission so we can also not impose tax on an area based on its urbanization.

Here, we are giving a method using which a fair imposition of environmental tax can be made. If M unit area is under consideration over which we need to impose environmental tax. Then, we firstly divide total area in n compartments $A_i (1 \leq i \leq n)$ such that each part of the i^{th} compartment emits GHGs in almost same rate. Let M_i be the area of the i^{th} compartment and α_i be the rate of emission of GHGs per unit area of A_i . We take α to be the critical value of rate of emission of GHGs per unit area. We will choose this value from a general inspection that area where economically lower class people reside has rate of emission of GHGs per unit area is less than α .

Now, if I_i denotes the environmental tax which should be imposed on area M_i , then I_i will be defined as

$$I_i = \begin{cases} (\alpha_i - \alpha)\beta & \text{if } \alpha_i \geq \alpha, \\ 0 & \text{if } \alpha_i \leq \alpha, \end{cases} \quad (2.1)$$

where β is a proportionality constant which we will calculate in next section.

A trivial question arises that how we will measure α_i from i^{th} compartment. Here we are suggesting two methods using which we can approximately recognize its value.

2.1 Method:1

Monitoring of air pollution was very costly in past days but recently a combined effort of “Respirall Living Sciences Startup” and “IIT Kanpur” has given a very pleasant result. They

have developed an instrument for measuring air pollution whose cost is 50 times less than that of previous (Sikrodia , 2017).

Thus now we can easily measure the value of α_i for M_i . A point must be noted here that M_i 's must be small for more precise results. This method may be unfair for the persons who are living in industrial area like Delhi but are not responsible for the emission of GHGs in such a large extent.

2.2 Method:2

In this method, firstly we record the standard value of rate of emission of GHGs from individual emitting bodies like industries, different vehicles, air conditioners, refrigerators e.t.c. then we impose environmental tax on the owners according to how much amount of their emitting bodies are emitting GHGs.

For applying this method we will divide the whole area in components according to their owners i.e. for different owners A_i 's should be different. This method will give more precise result than that of previous one.

By above discussed two methods we can divide the whole area into compartments and find the value of rate of emission of GHGs per unit area of individual compartment. Now a big question comes about how much environmental tax must be imposed such that in a prescribed time the concentration of GHGs reduces to some extent. To find out the answer of these important questions we have considered following mathematical model.

2.3 Mathematical Formulation

To study the dynamics of concentration of GHGs, we will use the theory of ordinary differential equations (Turchin , 2003). Here, we are considering that concentration of GHGs is constantly increasing excluding the increment due to human induced causes and we are also considering some natural depletion in its concentration. If we consider α_i to be the rate of emission of GHGs per unit area of the compartment A_i then the rate of emission of GHGs due to whole area $M(= \sum_{i=1}^n M_i)$ under consideration will be $\sum_{i=1}^n \alpha_i M_i$. If we impose I_i ($1 \leq i \leq n$) amount of tax on A_i then concentration of GHGs will reduce due to this tax and if we take δ , a proportionality constant, which is reduction in rate of emission of concentration of GHGs per unit tax then the reduction in concentration of GHGs on imposing tax on whole area will be $\delta \sum_{i=1}^n I_i$.

Hence, we are coming up with the equation:

$$\frac{dX}{dt} = Q_0 + \sum_{i=1}^n \alpha_i M_i - \gamma X - \delta \sum_{i=1}^n I_i; \quad (2.2)$$

which is a first order non homogeneous linear differential equation whose solution is given by

$$X = ce^{-\gamma t} + \frac{1}{\gamma} \left[Q_0 + \sum_{i=1}^n \alpha_i M_i - \delta \sum_{i=1}^n I_i \right], \quad (2.3)$$

where c is an arbitrary constant.

If initial concentration of GHGs is X_0 , i.e. at $t = 0$, $X = X_0$ then from equation ((2.3)), we find that

$$c = X_0 - \frac{1}{\gamma} \left[Q_0 + \sum_{i=1}^n \alpha_i M_i - \delta \sum_{i=1}^n I_i \right], \quad (2.4)$$

substituting this value of c in equation ((2.3)), we get

$$X = X_0 e^{-\gamma t} + \frac{1 - e^{-\gamma t}}{\gamma} \left[Q_0 + \sum_{i=1}^n \alpha_i M_i - \delta \sum_{i=1}^n I_i \right] \quad (2.5)$$

Equation ((2.5)) is giving the value of concentration of GHGs after time t if we are imposing a tax of amount I_i ($1 \leq i \leq n$) on i^{th} compartment A_i of the whole area under consideration.

Table:1 Nomenclature and abbreviation

X	Concentration of GHGs
Q_0	Emission rate of GHGs from natural sources and anthropogenic sources (excluding increment due to human being)
$M_i(1 \leq i \leq n)$	Area of i^{th} compartment
$\alpha_i(1 \leq i \leq n)$	Rate of emission of GHGs per unit area of A_i
γ	Natural depletion rate coefficient of GHGs
δ	Rate of emission of concentration of GHGs per unit tax
$I_i(1 \leq i \leq n)$	Environmental tax to be imposed on $M_i(1 \leq i \leq n)$
α	Critical Value of rate of emission of GHGs per unit area
β	Amount of environmental tax to be imposed on $M_i(1 \leq i \leq n)$ if the difference of $\alpha_i(1 \leq i \leq n)$ and α is unity.

3 Results

3.1 Persistence

Taking t tending towards infinity in equation ((2.5)), we get

$$\lim_{t \rightarrow \infty} X = \frac{Q_0 + \sum_{i=1}^n \alpha_i M_i - \delta \sum_{i=1}^n I_i}{\gamma} = \bar{X}$$

which shows that solution of our model ((2.2)) is bounded as well as persistent and independent of initial concentration of GHGs provided that

$$Q_0 + \sum_{i=1}^n \alpha_i M_i > \delta \sum_{i=1}^n I_i. \quad (3.1)$$

Also $\bar{X}$ will be an interior equilibrium point of model ((2.2)) which will exist if condition (3.5) is satisfied by parameters.

3.2 Evaluation of total tax to be imposed on prescribed area M

Equation ((2.5)) can also be written as

$$\sum_{i=1}^n I_i = \frac{1}{\delta} \left[Q_0 + \sum_{i=1}^n \alpha_i M_i - \frac{\gamma(X - X_0 e^{-\gamma t})}{(1 - e^{-\gamma t})} \right] = \xi \quad (\text{say}). \quad (3.2)$$

Equation ((3.2)) is giving the total amount of tax which should be imposed on area M to reduce the concentration of GHGs from X_0 to X in time t .

3.3 Distribution of tax on compartments

Equation ((3.2)) is giving the value of total tax which should be imposed on whole area M . By rearrangement of compartments, we can come up in a position that if $i < j$ then $\alpha_j \leq \alpha_i$ ($1 \leq i, j \leq n$) and if we are taking α to be the critical value of rate of emission of concentration GHGs per unit area then there will be some compartments which will emit GHGs in a rate greater than or equal to α . Let such type of compartment be m in number so we will have from equation ((2.1)) $I_i = 0$ for ($m < i \leq n$) and $I_i = (\alpha_i - \alpha)\beta \neq 0$ for ($1 \leq i \leq m$) so total tax $\sum_{i=1}^n I_i$ will be $\sum_{i=1}^m I_i$. Using this value in equation ((3.2)). We get

$$\sum_{i=1}^m (\alpha_i - \alpha)\beta = \xi,$$

this gives the value of " β " as follows:

$$\beta = \frac{\xi}{\sum_{i=1}^m (\alpha_i - \alpha)} = \frac{\xi}{\sum_{i=1}^m \alpha_i - m\alpha}, \quad (3.3)$$

where ξ is given by equation ((3.2)). Equation ((3.3)) is giving the value of proportionality constant β to reduce the value of concentration of GHGs from X_0 to X in time t .

4 Discussion

Here, we have conducted some numerical computation to check the feasibility of our analysis and to describe the method of computing environmental tax using some relevant hypothetical data set. We choose the values of parameters Q_0 (Misra and Verma, 2013), γ (Sharma and Dikshit, 2016) and δ as follows:

$$Q_0 = 5, \gamma = 0.27, \delta = 0.001, \quad (4.1)$$

values of α_i and M_i ($1 \leq i \leq n$) are given in table:2.

Now, firstly we will calculate the total amount of environmental tax which should be imposed on the total area under consideration i.e. $\sum_{i=1}^n (M_i) = 59$ unit area if the present concentration of GHGs is 400 unit and after 5 unit year we want it to be 200 unit.

From table:2 we get that 10 areas are emitting GHGs more than critical value and

$$\sum_{i=1}^{14} (M_i) = 59, \sum_{i=1}^{10} (\alpha_i) = 71, \sum_{i=1}^{14} (M_i \alpha_i) = 400, \sum_{i=1}^{10} (\alpha_i - \alpha) = 3$$

Table:2 Distribution of environmental tax

Compartment	Area M_i	α_i	$M_i\alpha_i$	if critical value of $\alpha = 4$ then $\alpha_i - \alpha$	I_i
A_1	8	10	80	6	7.1593×10^4
A_2	7	9	63	5	5.9661×10^4
A_3	5	9	45	5	5.9661×10^4
A_4	6	8	48	4	4.7729×10^4
A_5	5	7	35	3	3.5797×10^4
A_6	5	6	30	2	2.3864×10^4
A_7	4	6	24	2	2.3864×10^4
A_8	3	6	18	2	2.3864×10^4
A_9	4	5	20	1	1.1932×10^4
A_{10}	2	5	10	1	1.1932×10^4
A_{11}	3	4	12	0	0
A_{12}	3	3	9	-1	0
A_{13}	2	2	4	-2	0
A_{14}	2	1	2	-3	0
$\sum_{i=1}^{14} I_i = 3.6989 \times 10^4$					

Further, substituting all the values of parameters in equation ((3.2)), we get the total environmental tax

$$\sum_{i=1}^m I_i = 3.6989 \times 10^4 \quad (4.2)$$

Now, our major task is to distribute this tax between 14 areas according as they are emitting GHGs. So as described previously from equation ((3.3)), we find the value of β as

$$\beta = 1.1932 \times 10^4 \quad (4.3)$$

and hence according to formula described in equation ((2.1)) we can calculate the amount of environmental tax which should be imposed on i^{th} ($1 \leq i \leq n$) area and it is given in table:2.

Further, we can also see that the condition (3.5) of persistence as well as existence of equilibrium is also satisfied by these parameters.

5 Conclusion

Continuously increasing level of greenhouse gases in the atmosphere is the main cause behind the threat of global warming. If this trend of GHGs emission continues then the menace of climate change become more forbidding. Mitigation of the GHGs is very crucial factor because it has very bad impact on human population and ecological system nowadays. So, some major steps should be taken to improve the health of the ecosystem and minimize the concentration of GHGs in the atmosphere. It is evident that efforts are urgently needed to enhance everybody's awareness about what is happening around us. This should include positive and

eco-friendly actions by scientists, environmental biologist, researchers coupled with political will and sustained campaign to increase the public participation in the matter.

In this direction, we have suggested a simple method to impose environmental tax on the bodies which are emitting GHGs excessively and derived a formula relating concentrations of greenhouse gases, time and environmental tax from which we can calculate the amount of environmental tax which should be imposed on prescribed area to reduce the concentration of GHGs up to some extent in prescribed time and values of environmental tax also have been calculated for individual compartments.

It has been seen that power plants are one of the major emitter of GHGs. Imposing taxes on power plants will result an increment in the rates of their products, which will compel people to use eco-friendly sources like LED lights, solar panels etc.. Thus, if we are imposing tax on the owners of vehicles according as their vehicle is contributing in global warming they will use those vehicles which are less emitters of GHGs. Thus, our analysis reveals that imposition of environmental tax may give some positive results and can save ecosystem from the adverse effects of climate change.

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