

Impact of Pollutants in the Patchy Atmosphere Due to Mesoscale Wind under Multiple Point Sources

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ABSTRACT

Urban heat island (UHI) is caused due to the temperature difference between the city and its adjacent areas. Due to this UHI effect, mesoscale wind is created inside the city. Mostly the industries are set up adjacent to a city. These industries emit pollutants which reach the city and add up with those pollutants which are already being emitted due to the vehicles, biomass burning and other factors already present inside the city. In such a situation, it becomes necessary to study the overall impact of pollutants inside the city. Thus, we establish a model for the overall study of the pollutant's dispersion in a patchy atmosphere. The first region corresponds to a rural area where we assumed the pollutants to be emitted from an industry (a point source). These pollutants are transported to the second region (urban) due to advection and diffusion. In the second region, we considered that due to human activities the additional pollutants are emitted from multiple point sources. The pollutants in an urban area are dispersed in the atmosphere due to the mesoscale wind and local wind coming from rural area. The overall situation is defined by the two-dimensional advection-diffusion equations along with the appropriate boundary conditions. The overall analysis shows that the concentration level of pollutants inside the city is much higher as compared to its adjacent area. The mesoscale wind shapes the distribution of pollutants emitted from multiple point sources and the pollutants coming from rural area. Also, the pollutants coming from rural area contribute to a significant level in enhancing the concentration level inside the city.

Keywords: Urban heat island, mesoscale wind, point source, multiple point sources, variable wind profile and coefficient of diffusivity.

Mathematics Subject Classification Number: 97M10, 97Mxx, 91B76

1. INTRODUCTION

Air pollution is an alarming problem in today's scenario. In India, air pollution has become a national crisis due to low air quality in the urban area which has influenced the environment and has increased number of health issues like heart disease, lung cancer and respiratory diseases. Urban areas retain solar energy for a longer period due to dense population, huge buildings and industries, whereas rural area lose heat energy rapidly Rao (2014). Due to the heat created by the energy produced from the human activities, there is a temperature difference between the city (urban) and its adjacent

(rural) areas which creates the urban heat island (UHI) inside the city. Oke(1976) predicted that UHI becomes prominent under certain weather conditions (weak winds and cloudless skies).

There are factors like weather, geographic area, seasons, city formation and its function which are responsible for the formation of UHI. Later on, Kratzer (1956), Landsberg (1970) and Patterson (1969) made a significant contribution in underlying the concept of city climatology. Dorste et.al. (2018) in his paper focussed on the difference between the urban and rural wind dynamics which can be caused by the increase of surface roughness, surface heating due to UHI. It was concluded that the mean wind in cities can exceed the wind in rural surrounding. Under calm background conditions, the pollutants may be transported to the neighbouring cities through the dome flow which causes regional air pollution Fan et.al. (2017).

Chander (1968) found that the winds produced by urban heat island (UHI) tend to increase pollution gradient between city and its adjacent areas. Griffith (1970) pointed out that the knowledge of large-scale wind is not sufficient for air pollution forecast in city but the mesoscale wind plays a crucial role in shaping air pollution pattern inside the city. Dilley and Yen (1971) studied the distribution of air pollution from a line source under mesoscale wind. This shows that the urban heat island produces their own mesoscale winds which plays a crucial role in transportation of pollutants because the pollutants get trapped and therefore such a situation becomes a serious issue. In recent year, Agarwal and Tandon (2010) proposed a paper regarding the effect of mesoscale winds generated due to urban heat island effect on the dispersion of air pollutant under different atmospheric conditions.

The Gaussian plume model for turbulent dispersion air pollutants was widely used for air quality analysis and regulatory purposes Hinrichsen (1986), Wolfinger (1989). The limitation of the model of assuming the wind speed and eddy diffusivity to have no spatial and temporal variation made this model unsuitable in many situations. Hanna et.al. (1982) suggested that the Gaussian plume model was unsuitable or inappropriate for taking up the realistic situation of the turbulent dispersion near the surface of the earth where mean wind speed and eddy diffusivity are not constant. Several studies have been made by considering wind profile and eddy diffusivity in a two-dimensional advective diffusive equation to be a function of vertical height defined by power law profile Demuth (1978), Tirabassi (1989), Lin and Hildemann (1996).

In past, several studies have been made while considering the dispersion of pollutant in a patchy atmosphere. Shukla and Chauhan (1987) presented a model on a dispersion of reactive air pollutant in a two layered environment: effect of green belt, where the importance of greenbelt as a natural removal of pollutant was exhibited. In same way, Shukla et.al. (1992) proposed a model to study the effect of rain washout on the dispersion of pollutant in the atmosphere while considering a patchy atmosphere. The study revealed that the rain droplets remove the pollutants to a considerable level from the atmosphere.

In practical situation, it was observed that the industries which are set up in rural areas contribute a significant amount of air pollution to the nearby cities because the local winds transport these pollutants along the downwind direction towards the cities where these pollutants are added with the existing one emitted from number of point sources already present in the urban area. In case, if urban island heat effect (UHI) produces the mesoscale wind then all such pollutants get trapped and produce a huge impact on the environment making the urban air quality very poor and therefore become harmful for human life.

While considering such a situation, we proposed a model to study the dispersion of pollutant from a point source situated in rural area considered as the first region where wind profile and eddy

diffusivity are considered to vary along the vertical height and is defined as power law profile. These pollutants are transported along the downwind distance towards the urban area considered as a second region where there already exist multiple point sources emitting the air borne contamination. We assumed that due to the existence of urban heat island (UHI) there was a mesoscale wind produced and the pollutants coming from the rural area along with the contamination from the multiple point sources are transported in the atmosphere due to the mesoscale and local wind. The overall structure of the model is defined by the two-dimensional advective diffusion equation. The model assumed that due to the existence of greenbelt and water bodies the removal of pollutants take place. Further, different deposition velocity of pollutants is considered at ground level for both the region. The analytical approach is adopted to obtain the required solution with the help of separable of variable, power series and Green function.

2. MATHEMATICAL FORMULATION

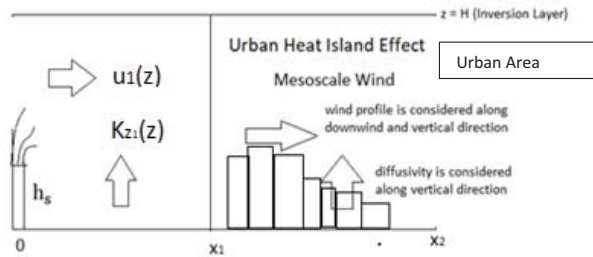


Figure: 1

In general, industrial areas are established in the rural area adjacent to the city where the pollutants are emitted and carried towards the urban area where already there exist pollutants which are being emitted due to different kind of human activities. During the daytime when there are maximum activities inside the city then the cumulative impact of pollutants in an urban area are needed to be studied accordingly.

Region-I: The region $0 \leq x \leq x_1$ as prescribed in figure 1 are considered as rural area where a continuous emission of pollutants takes place at height h_s from a point source (taken as chimney of an industry). In the region of interest, it was assumed that the mean concentration of pollutants is constant along the crosswind distance and hence the governing advective-diffusion equation is considered as two-dimensional partial differential equation.

$$u_1(z) \frac{\partial C_1}{\partial x} = \frac{\partial}{\partial z} \left(K_{z_1}(z) \frac{\partial C_1}{\partial z} \right) - \alpha_1 C_1 \quad (2.1)$$

where C_1 is the concentration of pollutants at any point (x, z) , $u_1(z)$ represent the wind speed along the downwind distance x , $K_{z_1}(z)$ represent the diffusivity coefficient along the vertical direction z , α_1 is the removal term which is considered to be constant in the prescribed region.

The pollutants are assumed to be dispersed within the mixing layer extended up to the height H above the ground $z = 0$. The movement of air pollutants is due to advection and diffusion process where the wind profile in the advection term and the diffusivity coefficient in the diffusion term are defined as power law profile as considered in the previous papers (Lin and Hildemann (1996), Agarwal and Joseph, 2010). Thus,

$$u_1(z) = u_H \left(\frac{z}{H} \right)^n \text{ and } K_{z_1}(z) = K_H \left(\frac{z}{H} \right)^m \quad (2.2)$$

where $m, n \geq 0$, u_H and K_H are wind speed and diffusivity coefficient at a reference height H .

For the simplicity of problem, we take $m = n = \frac{1}{2}$

Using equation (2.2), the equation (2.1) becomes

$$u_H \left(\frac{z}{H} \right)^{1/2} \frac{\partial C_1}{\partial x} = \frac{\partial}{\partial z} \left(K_H \left(\frac{z}{H} \right)^{1/2} \frac{\partial C_1}{\partial z} \right) - \alpha_1 C_1 \quad (2.3)$$

As the pollutants is emitted from a point source at a height h_s from the ground. Thus,

$$C_1(x, z) = \frac{Q_1 \delta(z-h_s)}{u} \quad \text{at } x = 0 \quad (2.4)$$

where Q_1 is the constant emission rate of pollutants, $\delta(\cdot)$ is the Dirac-delta function.

The pollutants so emitted from a point source at a height h_s from the ground are removed from the atmosphere when they come in contact with the ground due to absorption (dry deposition). Thus,

$$K_{z_1}(z) \frac{\partial C_1}{\partial z} = v_{d_1} C_1 \quad \text{at } z = 0 \quad (2.5)$$

where v_{d_1} represents the deposition velocity. The pollutants are confined within the mixing layer and cannot penetrate through the inversion layer so,

$$K_{z_1}(z) \frac{\partial C_1}{\partial z} = 0 \quad \text{at } z = H \quad (2.6)$$

Region-II: In the urban area $x_1 \leq x$ as shown in Figure-1, we assumed that due to the physical nature of various human activities inside the city there exists multiple sources which emits pollutants. These pollutants are assumed to be emitted at a constant rate and spread within the mixing layer. Along with these pollutants, the air pollution so carried from the region-I mixes with the pollutants from multiple sources without creating any chemical reaction. The pollutants are transported due to advection and diffusion within the city. In an urban area the heat island produces its own mesoscale wind which we assumed to take place. The urban heat island effect causes a rising of air above the centre of the island. The rising of air draws in a surface influx or air from the environment. In such a situation a thermally induced convective current is set up Dilley and Yen (1971). To understand mesoscale wind so formed under heat island effect we follow a reverse stagnation point flow for which

$$u_e \propto -x, \quad w_e \propto z \quad (2.7)$$

Equation (2.7) represent the surface influx and a rising current Dilley and Yen (1971). The large-scale wind coming from rural area and vertical diffusivity are too considered to follow the power law profile.

$$u_2(z) = u_H \left(\frac{z}{H} \right)^p \quad \text{and} \quad K_{z_2}(z) = K_H \left(\frac{z}{H} \right)^q \quad (2.8)$$

where $p, q \geq 0$ H is the inversion height which we considered same for both the region.

It was assumed that the horizontal mesoscale wind varies in the same manner as $u_2(z)$. The horizontal and vertical mesoscale wind can be written as suggested by Dilley and Yen (1971)

$$u_e = -ax \left(\frac{z}{H} \right)^r \quad \text{and} \quad w_e = \frac{az}{(r+1)} \left(\frac{z}{H} \right)^r \quad (2.9)$$

where $r \geq 0$ and a is proportionality constant.

Thus, the dispersion of pollutants under the influence of mesoscale and local wind and diffusivity is defined in the form of two-dimensional advective diffusion equation for the pollutant coming from region -I and from multiple point sources existed within the city.

$$(u_2 + u_e) \frac{\partial C_2}{\partial x} + w_e \frac{\partial C_2}{\partial z} = \frac{\partial}{\partial z} \left(K_{z_2}(z) \frac{\partial C_2}{\partial z} \right) - \alpha_2 C_2 \quad (2.10)$$

$$(u_2 + u_e) \frac{\partial C_3}{\partial x} + w_e \frac{\partial C_3}{\partial z} = \frac{\partial}{\partial z} \left(K_{z_2}(z) \frac{\partial C_3}{\partial z} \right) - \alpha_3 C_3 + \sum_{j=1}^n Q^j \delta(x - x_s^j) \delta(z - z_s^j) \quad (2.11)$$

Here C_2 corresponds to the concentration of pollutants coming from region -I and C_3 represents the concentration of pollutants from multiple point sources. α_2 and α_3 represents the constant removal

parameter for C_2 and C_3 due to the existence of green belt, water bodies in the city. Q^j represents the source strength of the source terms. $\delta(\cdot)$ represents the Dirac-delta function.

For the simplicity of problem, we take $p = q = r = \frac{1}{2}$

Using equations (2.8) and (2.9) the equations (2.10) and (2.11) becomes

$$(1 - ax) \left(\frac{z}{H} \right)^{1/2} \frac{\partial C_2}{\partial x} + \frac{2az}{3} \left(\frac{z}{H} \right)^{1/2} \frac{\partial C_2}{\partial z} = \frac{\partial}{\partial z} \left(K_H \left(\frac{z}{H} \right)^{1/2} \frac{\partial C_2}{\partial z} \right) - \alpha_2 C_2 \quad (2.12)$$

$$(1 - ax) \left(\frac{z}{H} \right)^{1/2} \frac{\partial C_3}{\partial x} + \frac{2az}{3} \left(\frac{z}{H} \right)^{1/2} \frac{\partial C_3}{\partial z} = \frac{\partial}{\partial z} \left(K_H \left(\frac{z}{H} \right)^{1/2} \frac{\partial C_3}{\partial z} \right) - \alpha_3 C_3 + \sum_{j=1}^n Q^j \delta(x - x_s^j) \delta(z - z_s^j) \quad (2.13)$$

The concentration level for $C_2(x, z)$ is assumed to be same as $C_1(x, z)$ at the partition level $x = x_1$ i.e.

$$C_1(x_1, z) = C_2(x_1, z). \quad (2.14)$$

The concentration level C_3 assumed to be zero as x reaches to infinity so,

$$C_3(x, z) = 0 \text{ as } x \rightarrow \infty \quad (2.15)$$

The pollutants are assumed to be removed when come in contact to the ground due to absorption. This can mathematically be represented as

$$K_{z_2}(z) \frac{\partial C_2}{\partial z} = v_{d_2} C_2 \quad z = 0 \quad (2.16)$$

$$K_{z_2}(z) \frac{\partial C_3}{\partial z} = v_{d_3} C_3 \quad z = 0 \quad (2.17)$$

v_{d_2} represent the deposition velocity of the pollutants whose concentration is C_2 and v_{d_3} is the deposition velocity of the pollutants whose concentration is C_3 .

The pollutants are confined within the mixing layer and cannot penetrate through the inversion layer so,

$$K_{z_2}(z) \frac{\partial C_2}{\partial z} = 0 \quad \text{at } z = H \quad (2.18)$$

$$K_{z_2}(z) \frac{\partial C_3}{\partial z} = 0 \quad \text{at } z = H \quad (2.19)$$

3. METHOD OF SOLUTION

The solution corresponding to partial differential equation along the boundary conditions in first and second patch respectively are need to be obtained to analyse the behaviour and impact of the pollution on the environment. We use the analytical approach which was previously adopted in several papers like Shukla et.al. (1992), Agarwal and Shukla (2002), Lin and Hildemann (1996) to find the solution. Initially we convert the equations in dimensionless form by defining the dimensionless quantities as

$$\text{For Region- I: } \bar{x} = \frac{K_H x}{u_H H^2}, \quad \bar{h}_s = \frac{h_s}{H}, \quad \bar{z} = \frac{z}{H}, \quad \bar{C}_1 = \frac{u_H H^2}{Q_1} C_1, \quad \bar{\alpha}_1 = \frac{H^2}{K_H} \alpha_1 \quad (3.1)$$

Such procedure was previously adopted in the paper Arora et. al. (1991), Shukla et.al. (1992), Joseph and Agarwal (2020).

The equation (2.3) after dropping bar for convenience becomes

$$z^{1/2} \frac{\partial C_1}{\partial x} = \frac{\partial}{\partial z} \left(z^{1/2} \frac{\partial C_1}{\partial z} \right) - \alpha_1 C_1 \quad (3.2)$$

The boundary conditions (2.4), (2.5) and (2.6) becomes after dropping bar for convenience

$$C_1(x, z) = \frac{Q_1 \delta(z - h_s)}{z^{1/2}} \quad \text{at } x = 0 \quad (3.3)$$

$$z^{1/2} \frac{\partial C_1}{\partial z} = N_1 C_1 \quad \text{at } z = 0 \quad (3.4)$$

$$\text{where } N_1 = \frac{v_{d_1} H}{K_H}$$

$$z^{1/2} \frac{\partial C_1}{\partial z} = 0 \quad \text{at } z = 1 \quad (3.5)$$

The corresponding solution of the equation (3.2) along with the boundary conditions (3.3) – (3.5) by using the separation of variable technique and power series is represented as

$$C_1(x, z) = \sum_{n=1}^{\infty} \frac{M_n(h_s)}{\int_0^1 z^{1/2} M_n^2(z) dz} \exp(-\lambda_n^2 x) M_n(z) \quad (3.6)$$

$$\text{where } M_n(z) = \left(1 + \frac{4}{3} M z^{3/2} - \frac{4}{3} N z^2 + \frac{16}{45} M^2 z^3 \dots\right) + N_1 \left(2 z^{1/2} + \frac{4}{3} M z^2 - \frac{8}{5} N z^{5/2} \dots\right) \quad (3.7)$$

$$\text{such that } M = \frac{\alpha_1}{2} \text{ and } N = \frac{\lambda^2}{4}. \quad (3.8)$$

$$\text{Here } N_1 = \frac{v_{d1} H}{K_H} \text{ and eigenvalues of } \lambda \text{ are obtained from the equation} \quad (3.8a)$$

$$\left(2M - \frac{8}{3} N + \frac{48}{45} M^2 \dots\right) + N_1 \left(1 + \frac{8}{3} M - 4N \dots\right) = 0 \quad (3.9)$$

For Region II: The dimensionless quantities corresponding to region -II is as follows

$$\bar{x} = \frac{K_H x}{u_H H^2}, \quad \bar{h}_s = \frac{h_s}{H}, \quad \bar{z} = \frac{z}{H}, \quad \bar{C}_2 = \frac{u_H H^2}{Q_2} C_2, \quad \bar{\alpha}_2 = \frac{H^2}{K_H} \alpha_2, \quad \bar{C}_3 = \frac{u_H H^2}{Q} C_3, \quad \bar{\alpha}_3 = \frac{H^2}{K_H} \alpha_3, \quad \bar{Q}^j = \frac{Q^j}{Q}, \quad \delta(\bar{x} - \bar{x}_s^j) = \frac{u_H H^2}{K_H} \delta(x - x_s^j), \quad \delta(\bar{z} - \bar{z}_s^j) = H \delta(z - z_s^j) \quad (3.10)$$

The equation (2.12) and (2.13) becomes after dropping bar for convenience

$$(1 - \beta x) z^{1/2} \frac{\partial C_2}{\partial x} + \frac{2\alpha z}{3} z^{1/2} \frac{\partial C_2}{\partial z} = \frac{\partial}{\partial z} \left(z^{1/2} \frac{\partial C_2}{\partial z} \right) - \alpha_2 C_2 \quad (3.11)$$

$$(1 - \beta x) z^{1/2} \frac{\partial C_3}{\partial x} + \frac{2\alpha z}{3} z^{1/2} \frac{\partial C_3}{\partial z} = \frac{\partial}{\partial z} \left(z^{1/2} \frac{\partial C_3}{\partial z} \right) - \alpha_3 C_3 + \sum_{j=1}^n Q^j \delta(x - x_s^j) \delta(z - z_s^j) \quad (3.12)$$

$$\text{where } \beta = \frac{aH^2}{K_H} \quad (3.13)$$

The boundary conditions (2.14) – (2.19) becomes after dropping bar for convenience

$$C_1(x_1, z) = C_2(x_1, z) \quad (3.14)$$

$$C_3(x, z) = 0 \text{ as } x \rightarrow \infty \quad (3.15)$$

$$z^{1/2} \frac{\partial C_2}{\partial z} = N_2 C_2 \quad z = 0 \quad (3.16)$$

$$z^{1/2} \frac{\partial C_3}{\partial z} = N_3 C_3 \quad z = 0 \quad (3.17)$$

$$\text{where } N_2 = \frac{v_{d2} H}{K_H} \text{ and } N_3 = \frac{v_{d3} H}{K_H} \quad (3.18)$$

$$z^{1/2} \frac{\partial C_2}{\partial z} = 0 \quad \text{at } z = 1 \quad (3.19)$$

$$z^{1/2} \frac{\partial C_3}{\partial z} = 0 \quad \text{at } z = 1 \quad (3.20)$$

The solution of equation (3.11) along with boundary conditions (3.14), (3.16) and (3.19) is obtained by the use of separation of variable technique and power series and is represented as

$$C_2(x, z) = \sum_{i=1}^{\infty} \sum_{j=1}^{\infty} \frac{M_j(h_s)}{\int_0^1 z^{1/2} M_j^2(z) dz \int_0^1 R_i^2(z) dz} (1 - \beta x)^{\frac{p_i^2}{\beta}} \frac{1}{(1 - \beta x_1)^{\frac{p_i^2}{\beta}}} \exp(-\lambda_j^2 x) \int_0^1 M_j(z) R_j(z) dz R_i(z) \quad (3.21)$$

$$\text{where } R_i(z) = (1 + .66\alpha_2 z^{3/2} - .33p^2 z^2 + .088\alpha_2^2 z^3 + .06349\beta\alpha_2 z^{7/2} - .06349p^2\alpha_2 z^{7/2} - .03024p^2\alpha_2 z^{7/2} \dots) + N_1 (2z^{1/2} + .66\alpha_2 z^2 - .133\beta z^{5/2} - .4p^2 z^{5/2} + .06349\alpha_2^2 z^{7/2} \dots) \quad (3.22)$$

The eigen values of p_i^2 are obtained from the equation

$$(.99\alpha_2 - .66p^2 + .264\alpha_2^2 + .22\alpha_2\beta - .22p^2\alpha_2 - .1058p^2\alpha_2 - .12696\beta p^2 \dots) + N_2 (1 + 1.32\alpha_2 - .3325\beta + .222\alpha_2^2 \dots) = 0 \quad (3.33)$$

The concept of Green's function which was previously used to find the analytical solution in a paper Agarwal & Joseph (2010), is adopted to find the solution of equation (3.12) along with the boundary conditions (3.15), (3.17) and (3.20). The obtained solution is represented as

$$C_3(x, z) = \sum_{j=1}^n Q^j \left\{ \sum_{i=1}^{\infty} \frac{M_i'(z) M_i'(z_s^j)}{\int_0^1 z^{1/2} (M_i'(z))^2 dz} \frac{1}{(1 - \beta x_1)^{\frac{\eta_i^2}{\beta}}} \left(1 - \beta(x + x_1 - x_s^j) \right)^{\frac{\eta_i^2}{\beta}} \right\} \quad (3.34)$$

where $M_i'(z) = (1 + .66\alpha_2 z^{3/2} - .33\lambda^2 z^2 + .088\alpha_2^2 z^3 + .06349\beta\alpha_2 z^{7/2} - .06349\lambda^2\alpha_2 z^{7/2} - .03024\lambda^2\alpha_2 z^{7/2} \dots) + N_1(2z^{1/2} + .66\alpha_2 z^2 - .133\beta z^{5/2} - .4\lambda^2 z^{5/2} + .06349\alpha_2^2 z^{7/2} \dots)$ (3.35)

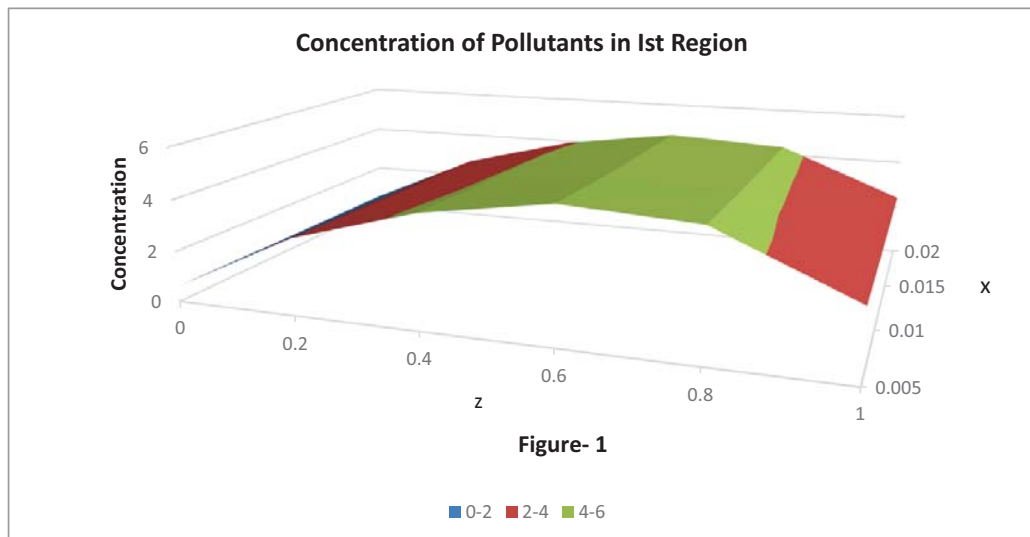
The eigen values of λ_i^2 are obtained from the equation

$$(.99\alpha_2 - .66\lambda^2 + .264\alpha_2^2 + .22\alpha_2\beta - .22\lambda^2\alpha_2 - .1058\lambda^2\alpha_2 - .12696\beta\lambda^2 \dots) + N_3(1 + 1.32\alpha_2 - .3325\beta + .222\alpha_2^2 \dots) = 0 \quad (3.36)$$

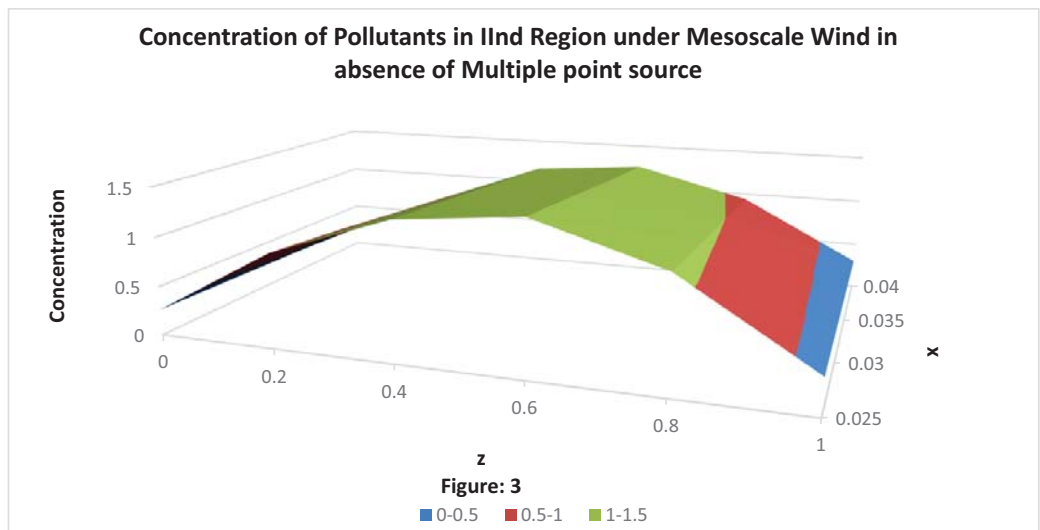
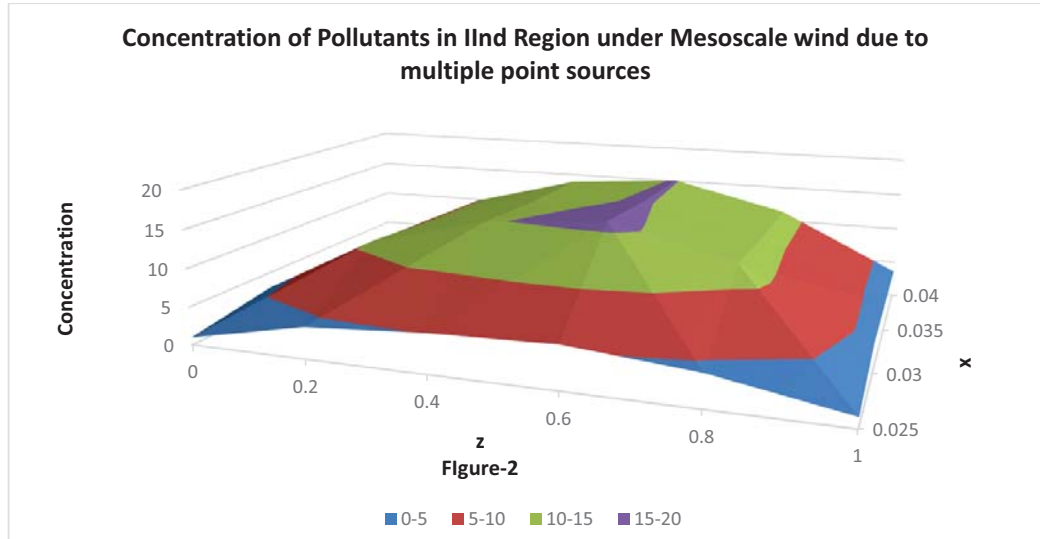
4. RESULT AND DISCUSSION

The analytical solution for the computation of concentration of pollutant so obtained in the form of equation (3.6) for region -I (rural region) and equations (3.21) and (3.34) for region -II (urban region) are analysed graphically by considering a set of suitable parameter values. The prescribed region-I is extended upto $x = .02$ along dimensionless downwind distance from the point source located at $x = 0$ for a dimensionless vertical height $z = .04$. The second region so considered as urban region is assumed to be started from $x = .02$. In urban area we have taken two different point sources located at different positions (x, z) as $(.025, 0)$ and $(.03, .04)$. The inversion height is taken at 1000m or $z = 1$. We considered the removal parameter in region-I to be 2.77 (Shukla et.al. 1988) and for region -II to be 1.77. The different values of removal parameters are considered with a thought that the availability of greenbelt and water resources are much higher in rural area as compared to urban area. The deposition velocity of pollutants from different sources are considered to be different due to surface roughness and therefore we considered v_{d1} as 1 cm/s and $v_{d2} = v_{d3} = 2$ cm/s (Shukla et.al. 1988). Thus, the parameter N_1, N_2, N_3 defined in equation (3.8a) and (3.18) respectively should be different for the both the patches and therefore, we have taken the values of $N_1 = 2, N_2 = N_3 = 4$. Further in region -II the value of Q^j is taken to be 1 for the simplicity. As the mesoscale wind exists in urban region so in this regard, we take $\beta = 10$.

The overall concentration level in both the patches are plotted in figure 1 -3 respectively. From figure 1 it was observed that the concentration level is prominent in the rural area especially, close to the source. The contamination level increases along the vertical height for upto $z = 0.6$ and then decreases. These pollutants which entered in second patch accumulates with the pollutants from other two sources

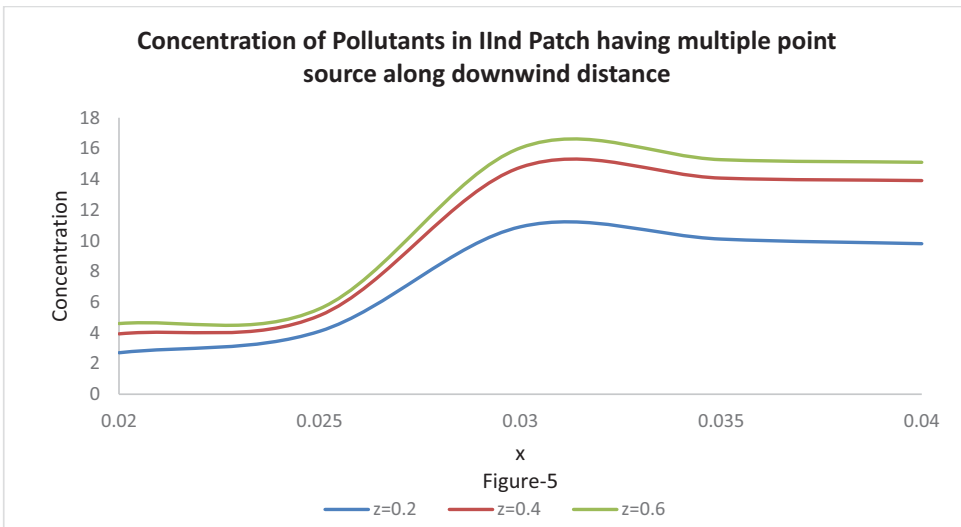
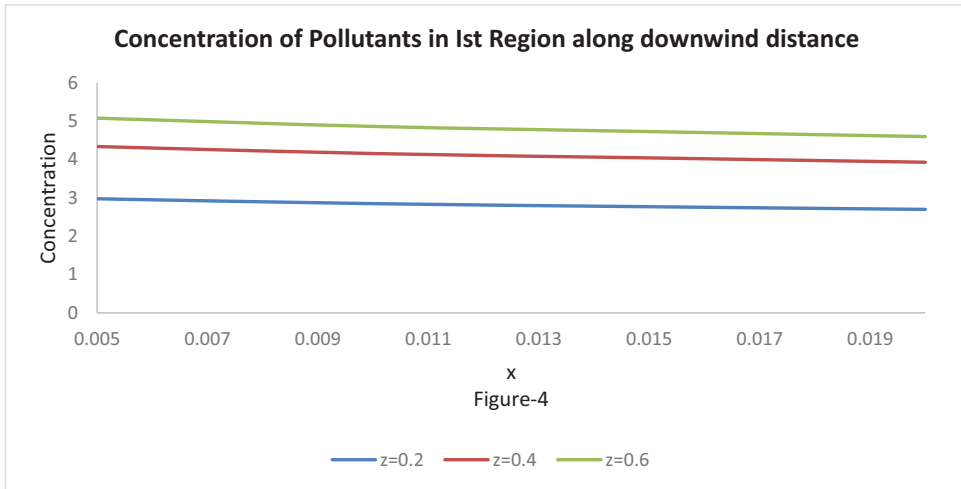


already present and hence show a remarkable rise in the concentration level as seen in figure 2. This highly rated concentration of pollutants exists throughout the region so considered.

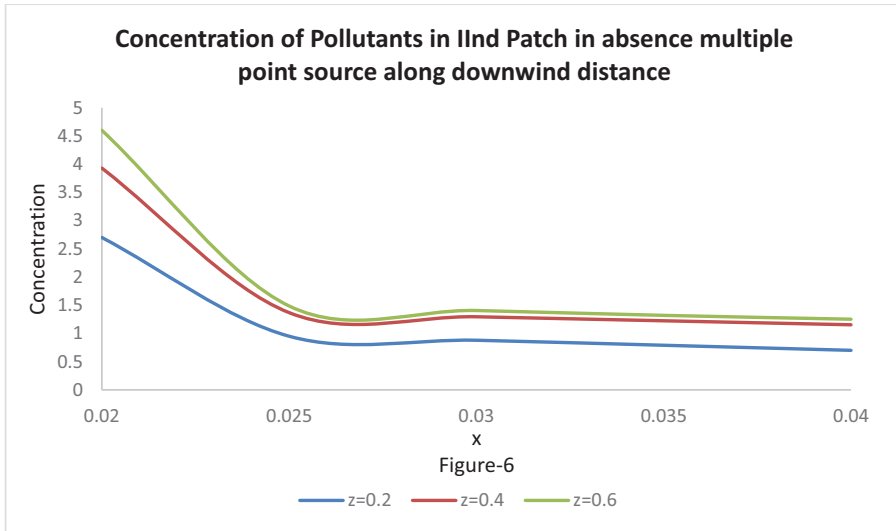


The behaviour of distribution of pollutants in figure 2 reveals that along the vertical height the concentration level increases up to $z=0.6$ and then decreases. The most highly densely polluted region shown in the second graph is between $0.03 \leq x \leq 0.04$ along downwind distance. In special case, we assumed the absence of multiple point sources and tried to find the impact of pollutants in an urban area which are coming from rural area. This particular case was depicted in figure-3. On comparing the two figures 2 and 3 it was found that the concentration level in figure-3 is quite low as compared to the level shown in figure -2. Secondly, the distribution of pollutant in figure -3 indicates that the concentration level decreases along the downwind distance and becomes almost negligible at $x=0.04$,

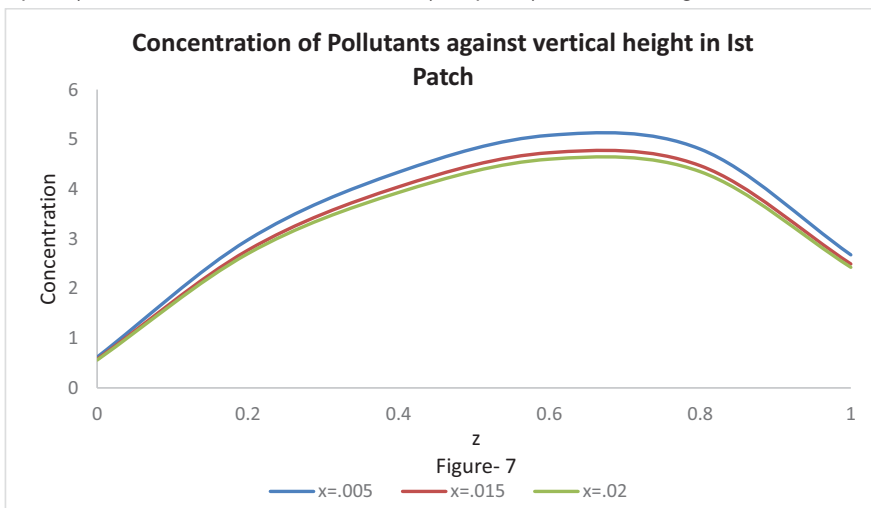
whereas in figure-2, the level of concentration does not continuously decrease along the downwind distance. The magnitude of the concentration sustains to a higher level in entire region.

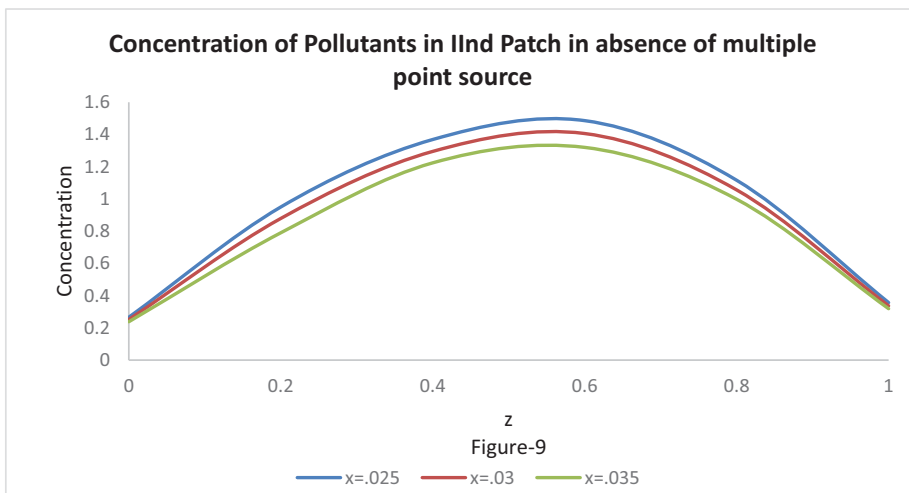
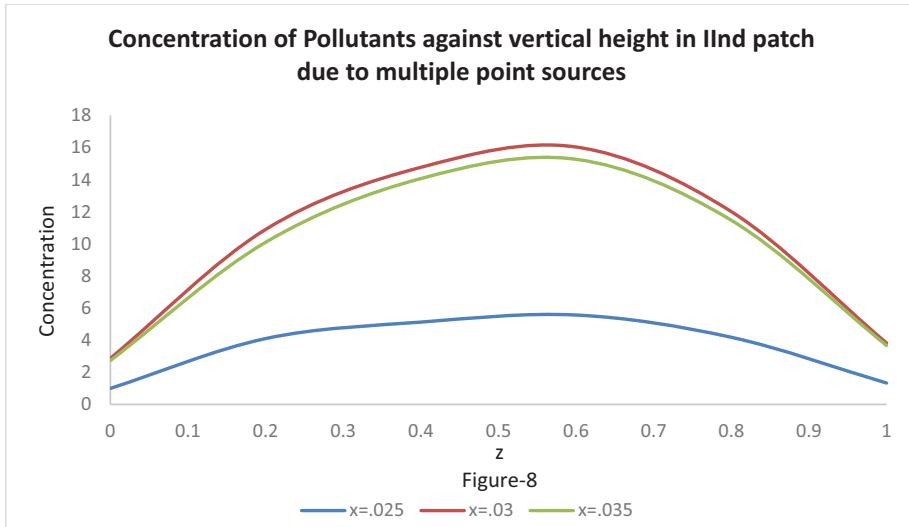


The dispersion of pollutants along the downwind distance for both patches are being exhibited in figures 4-6 respectively. It was noticed that the concentration level in the first patch which decreases slowly along the downwind distance persist to a considerable significant level at the boundary. Whereas in the second patch due to the existence of point source at $x=0.025$ and $x=0.03$ respectively the concentration level increases up to .03 and then decreases with a very slow rate. It also indicates that pollutants accumulate in the prescribed second region and cannot be dispersed rapidly along downwind distance. This shows the existence of high level of air pollutants in the overall region especially near the source term which is harmful for the environment. In case, when no source exists in the second patch, then the concentration level of the pollutant coming from rural area decreases along the downwind distance and reaches to a considerably lower level as shown in figure-6.

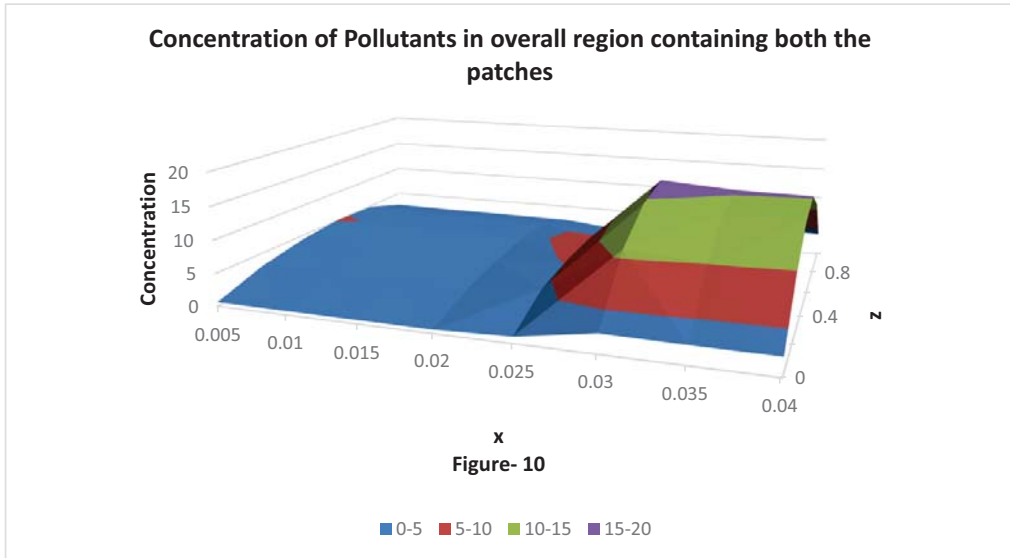


The variation of the concentration of pollutants against the vertical height can be seen in figures 7-9 respectively, where a similar trend regarding the dispersion of pollutants in both the patches are observed. The concentration level increases from the ground level to a height of $z = 0.6$ after which it starts decreasing and continued up to the inversion layer. The only difference is regarding the magnitude of the concentration where the second patch having multiple point sources contained a high level of concentration in comparison to the first patch. The absence of multiple point source in second patch shows that the only contribution of contamination in the second patch is because of the pollutants coming from the rural area (first patch) and hence, the concentration level of pollutants in urban area (second patch) is much lower than the rural area (first patch) as shown in figure 9.

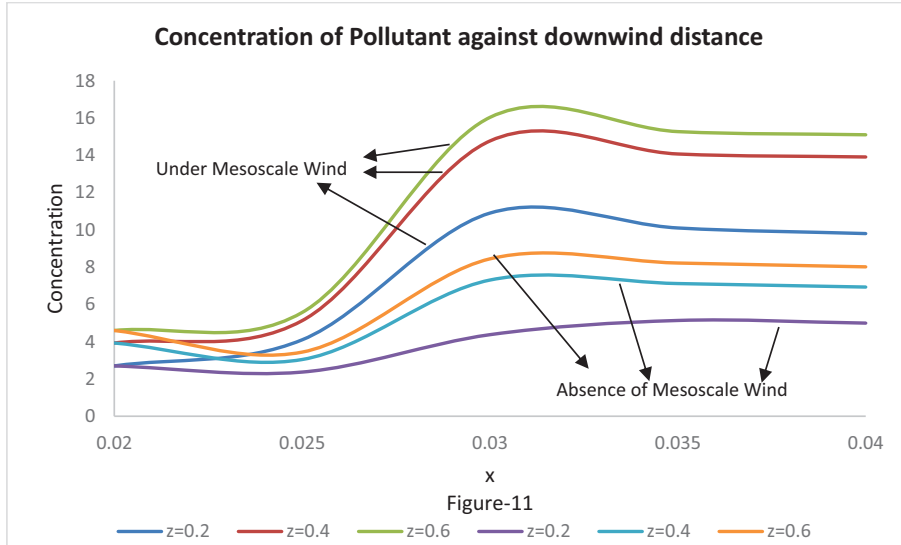




The behaviour of the dispersion of pollutants in the overall region containing both patches have been plotted as shown in figure -10. It was observed that the level of concentration increases tremendously in the second patch due to the existence of multiple point sources in presence of mesoscale wind. The pollutants coming from rural area also contributes significantly in raising the concentration level in an urban area. The mesoscale wind traps the pollutants and hence the concentration persists to a higher level inside the city (second patch). This situation adversely affects the environment of an urban area.



At last, a comparative analysis was made regarding the dispersion of pollutants while considering the presence and absence of the mesoscale wind. In absence of mesoscale wind, the local wind coming from rural area are responsible for the transportation of pollutants in an urban area. It was noticed that the level of concentration is much higher in the presence of mesoscale wind along the downwind distance as shown in figure -11. Thus, the need of removal mechanism becomes necessary to control the contamination level in an urban area during the presence of mesoscale wind.



5. CONCLUSION

A two- dimensional mathematical model has been proposed for a patchy atmosphere where we considered that a point source exist in the first patch considered as rural area. The pollutants emitted from the point source are transported in the atmosphere due to variable wind profile and diffusivity. The

variable wind profile and diffusivity are considered as height dependent and defined as power law profile. These pollutants finally, enter in the second patch considered as city (urban area) where there already exist multiple point sources. We considered that these pollutants are transported not only by local wind coming from rural area but also by mesoscale wind generated due to the urban heat island effect which is caused by the temperature difference between urban and rural area.

The overall process is analytically studied by defining a two-dimensional advection diffusion equation in both the patches with suitable boundary conditions. In both the region we considered that pollutants are being removed when come in contact with ground due to the deposition velocity as well as due to the green belt and water bodies which were considered in the form of constant removal parameter. Considering a practical situation, we have taken different magnitude corresponding to removal parameter for individual patches.

The results so obtained exhibits that the concentration of pollutants along the vertical height increases up to a height of $z = 0.6$ and then decreases. This trend was observed in both the patches. The GLC concentration in the rural area decreases with the increase of downwind distance but in case of urban area the concentration level increases up to $x = 0.03$, after which it starts decreasing slowly but still persist to a considerable level. This type of behaviour is due to the location of point sources inside the city, further from figure-11, it was concluded that the presence of mesoscale wind enhances the concentration level as the pollutants are being trapped and cannot be dispersed along the downwind distance.

The presence of multiple point sources also contributes in aggravating the GLC concentration. The figure 2 and 3 revealed that there is a huge difference in the concentration level while considering the presence and absence of multiple point sources. The presence of multiple point sources inside the city aggravates the concentration level. This calls for the reduce in number of human activities, in urban area which causes the production of air pollution. The overall picture of the distribution of pollutant as shown in figure -10 indicates that in comparison to rural area the concentration level of pollutants in urban area is very high and persist in overall second patch. These conditions require an attention for necessary action to be taken for decreasing the concentration of pollutants in urban area. This can be made possible by introducing the artificial removal mechanism. Finally, it was found that the pollutants coming from rural area contributes to a significant level in enhancing the concentration level in an urban area. This mean that there should be a check on the establishment of large number of industries near or adjacent to the urban area as well as monitoring system is necessary to be established regarding the functioning of the industries.

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