

Predictive Power of Stochastic Mortality Models for Different Age Groups: An Analysis of Developing and Developed Countries

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ABSTRACT

Mortality modelling helps in evaluating longevity and mortality risks by actuaries and demographers. The performance of mortality models strongly depends on the different salient features observed in different countries. In this paper, a comparative study of the Lee-Carter (LC) model, Age-Period-Cohort (APC) model and Cairns-Blake-Dowd (CBD) stochastic mortality models is presented for different age groups (0 to 4 years, 5 to 60 years and above 60 years) from developed and developing countries. The developed countries are France, Japan and Canada, and the developing countries are Ukraine and Taiwan. The forecast accuracy are evaluated using RMSE, MAE, MPE and MAPE metrics. Results indicate that models that are best for some age groups for developing countries might not be appropriate for developed ones since the evolution of each country's mortality rates is different from one another. Yet, for some ages, the same mortality model can be used to project mortality forecasts for both developing and developed countries. For higher ages (75+), the CBD model is found to outperform the other models across all countries.

Keywords: Mortality forecasting; Age-Period-Cohort model; Cairns-Blake-Dowd model; Lee-Carter model; Forecast Accuracy

Mathematics Subject Classification: 62P05

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1. INTRODUCTION

Human mortality modelling and forecasting has always been an interesting and crucial topic of study since the earliest years, among demographers, insurance mathematicians and actuaries. With the evolution of a number of mortality models, death can be better predicted nowadays. In fact, mortality modelling is considered to be the most dominant element in pricing and reserving of insurance products and pensions. Policymakers depend heavily on convenient mortality forecasts to determine pension or assurance benefits and make decisions regarding public pension reforms. Besides, modelling of mortality helps to quantify longevity risks, mortality risks and other risks which may massively affect human life.

Mortality or longevity risks are in fact the risks that actual mortality rates diverge from the expected mortality rates, which as a result largely influence reserves. In recent years, with improvements observed in mortality, longevity risk is perceived as being one of the most persistent financial risks for pension and public and private annuity providers. While some models have been designed to predict future evolution of mortality patterns, other mortality models have been formulated and explored to

mitigate the above-mentioned risks, especially for the pricing of longevity-mortality linked derivatives. These derivatives are simply financial securities which hedge against a group of insured individuals or pensioners who are exposed to that specific risk. However, other mortality models have been designed only for mortality forecasting.

Furthermore, mortality varies differently in countries which have a different development level. With time, distinct mortality and demographic features are also evolving across countries. Accessing mortality trends is a fundamental issue in actuarial modelling. Although life expectancies and mortality projections are computed and published by some firms in some countries, actuaries continue to design and explore new mortality models for better mortality modelling and reasonable pricing of securities. These demographic quantities will certainly be more accurate than the ones computed by the national agencies, and consequently help financial institutions deal with demographic risks.

Stochastic mortality modelling is of primary importance to the government and to the insurance companies. The 2-factor mortality model developed by Lee and Carter (1992) became one of the most dominant approach to stochastic mortality modelling in the demographic literature. The Lee-Carter (LC) model is undoubtedly the leading mortality model regarding modelling at all ages as stated by Deaton and Paxson (2004), as it does not require to have prior information regarding medical, behavioural or social changes on mortality. It is therefore computationally simple to implement. LC model has been extensively used in the literature for analysing mortality from different countries. For instance, several studies [Booth et al. (2006), Brouhns et al. (2002) and Chavhan and Shinde (2016)] applied the Lee-Carter model to both economically developed and developing countries.

Mortality trends are said to vary with time, and these changes are often associated with external factors such as gender, occupation, education and lifestyle. The birth cohort effect which refers to individuals born in the same year, the age and the period effects are identified to be the three main parameters. Evidences conducted by Willets (2004) show that people born in the same time frame are assumed to experience same patterns in mortality. Currie (2006) introduced the Age-Period-Cohort model (APC) to model mortality. It assumes that age, time and cohort parameters are independent in their effects on mortality. Theoretically, cohort models are believed to perform better than period models in terms of forecasting. Yet in practice, this is not always observed due to the restrictions caused by some factors. APC is useful when it comes to describing the past but less useful in forecasting as Booth and Tickle (2008) reported. This may be due to the large number of parameters which result in over-parameterisation. Moreover, models that fit historical data well do not necessarily require to produce adequate forecasts. Although cohort effects are relatively lower than period effects, nevertheless for some countries, they are clearly visible. Hunt and Villegas (2015) showed that individuals born in European countries between 1920 and 1940 experience significant improvements in mortality rates. In UK, for instance, lower mortality rates had been observed at adult ages for those born in the 1925-1944 relative to those born a decade before, mainly because of the aftermaths of war which had a great impact on the previous generation, as per Willets (2004). Cohort effects were also identified in the French data by Wilmoth (1990) and Japanese data by Willets (2004). The empirical findings suggest that the performance of the APC model depend on the population being studied.

Cairns et al. (2006) designed the Cairns-Blake-Dowd (CBD) model to model the human mortality at higher ages, essentially for pricing longevity bonds. Unlike LC and APC, the CBD model uses the mortality rate rather than death rate. Period effects are taken into account in the CBD model. They may come from external factors that equally affect all age groups at a particular calendar time. It could arise from a range of environmental, social and economic factors e.g. war, famine and economic crisis. CBD is different from LC and APC in the sense that it does not really require parameter constraints since it is fully identified, that is including additional data to the existing ones will hardly affect parameters which have already been estimated beforehand. This model best suits data in which the presence of volatility is felt, which makes it among the most appropriate models in projecting future mortality for higher ages. Other strengths of this mortality model are that it allows for different improvements for different ages over different periods. However, CBD also is associated with some undesirable features when modelling mortality. For instance, LC model is better able than CBD model to capture minor non-linearities in mortality curve. In addition, it does not incorporate any cohort effects as in the APC model. Maccheroni and Nocito (2017) performed a CBD analysis exclusively for ages between 57 and 90 for the Italian population. They concluded that CBD works well prevalently for ages above 75+. In addition, the model does not really work well over a relatively short time period which has seen a generally stable pattern of improvements in life expectancy. We suspect that the performance of the CBD model for different ages and periods will be highly dependent on the mortality pattern of the population being studied.

Several studies have been found employing the LC, APC and CBD models to fit and forecast mortality for different countries. The empirical literature suggests that their performance highly depends on the mortality pattern of the underlying population. The mortality pattern is different for different age groups and countries with different economic status. The purpose of this study is to compare the LC, APC and CBD mortality forecasts for different age groups across countries with different economic levels. The models are employed on mortality data of three developed countries (France, Japan and Canada) and two developing countries (Ukraine and Taiwan). Each population is split into three age-groups: ages 0 to 4 years old, 5 to 60 years old and above 60 years old. The mortality forecasts are assessed using the RMSE, MAE, MPE and MAPE metrics and conclusions are drawn from the results obtained.

The rest of this paper is classified into the following sections. Section 2 defines the mortality models employed. Section 3 details the different datasets under consideration. Section 4 presents the application of the mortality models for each dataset and the accuracy metrics for the forecasting performance. The findings of the analysis are discussed in Section 5, and conclusions are drawn in Section 6.

2. MODEL SPECIFICATION

This section describes the three mortality models which are employed in this paper: the Lee-Carter (LC) model, the Age-Period-Cohort (APC) model and the Cairns-Blake-Dowd (CBD) model.

2.1 The Lee-Carter (LC) Model

The stochastic Lee-Carter model is decomposed into the age-specific parameters and time-trend index as shown below:

$$\ln(m_{x,t}) = \alpha_x + \beta_x^{(1)} k_t^{(1)} + \varepsilon_{x,t} \quad \varepsilon_{x,t} \sim N(0, \sigma^2)$$

where α_x denotes the general age pattern of the log mortality rates averaged across years, and $\beta_x^{(1)}$ represents the rate of change in the central rate of mortality with respect to changes in $k_t^{(1)}$. The mortality index, $k_t^{(1)}$, exhibits the period effect which is the relationship between time-dependent events and mortality rates. The error term, $\varepsilon_{x,t}$ accounts for the random historical fluctuations not captured by the model. It is assumed that $\varepsilon_{x,t}$ are independent and identically distributed Gaussian random variables with mean 0 and variance σ^2 .

The parameters of the Lee-Carter model can be estimated through matrix operations using the Singular Value Decomposition (SVD) proposed by Lee and Carter. The basic Ordinary Least Squares (OLS) method cannot be used to fit the model. In literature, alternative methods are proposed for parameter estimation whereby the method of Maximum Likelihood Estimation and Weighted Least Squares technique (WLS) were implemented by Wilmoth (1993). Brouhns et al. (2002) and Renshaw and Haberman (2006) employed the method of Generalized Linear Models.

Since there is an issue of identification in this parametrisation which arises from the fact that one parameter estimate has more than one solution, it is important to ensure uniqueness of the parameter estimates. Hence, to implement the SVD procedure, two normalizing conditions are introduced which tackle this identification problem:

$$\sum_t k_t^{(1)} = 0 \text{ and } \sum_x \beta_x^{(1)} = 1$$

Normally, ARIMA models, in particular ARIMA (0,1,0) with drift has been used almost exclusively to model $k_t^{(1)}$ which is as follows:

$$k_t^{(1)} = k_{t-1}^{(1)} + \mu + e_t$$

whereby, μ represents the drift parameter, e_t represents the error term, which is a gaussian white noise, $e_t \sim N(0, \sigma_w^2)$.

The values of $\widehat{k}_t^{(1)}$ are then plugged into the original Lee-Carter model to obtain the forecasted mortality rates for the data under consideration.

2.2 The Age-Period-Cohort (APC) Model

The classical log-linear APC model formulated by Currie (2006), may be written as follows:

$$\ln(m_{x,t}) = \alpha_x + k_t^{(1)} + \gamma_{t-x} + \varepsilon_{x,t}$$

The first two parameters of the APC, α_x and $k_t^{(1)}$, share the same explanation as the parameters of the Lee-Carter model. The parameter γ_{t-x} relates to the general level of mortality for cohort born in the year $t - x$.

No smoothness conditions will be imposed in our analysis. Different scholars apply different methods to estimate APC parameters. The parameters of the APC model will be estimated using the SVD approach in our case in which α_x is firstly estimated as follows:

$$\hat{\alpha}_x = \frac{1}{t_n - t_1 + 1} \sum_{t=t_1}^{t_n} \ln(\hat{m}_{x,t})$$

The remaining two parameters, $k_t^{(1)}$ and γ_{t-x} are then estimated following the parameter constraints imposed to avoid any identification problems:

$$\sum_t k_t^{(1)} = 0 \text{ and } \sum_{x,t} \gamma_{t-x} = 0.$$

After obtaining the fitted parameters of $\hat{k}_t^{(1)}$ and $\hat{\gamma}_{t-x}$, univariate time series models are used for modelling and forecasting. It is assumed that the cohort effect follows a univariate ARIMA process which is independent of the period effect.

We shall follow the steps of Renshaw and Haberman (2006) and apply an ARIMA (1,1,0) with drift to obtain the forecasted values of the cohort effect, as described below:

$$\gamma_{t-x} = \phi \gamma_{t-x-1} + \mu + e_t$$

The forecasted cohort effects shall then be plugged into the original APC formulation to obtain the forecasted mortality rates for the data under consideration.

2.3 The Cairns-Blake-Dowd (CBD) Model

The original Cairns-Blake-Dowd (CBD) model is formulated as follows:

$$\text{logit}(q_{x,t}) = k_t^{(1)} + k_t^{(2)}(x - \bar{x})$$

As mentioned earlier, $k_t^{(1)}$ represents the mortality index, while $k_t^{(2)}$ which is the slope of the CBD model represents improvements in mortality. Each age is affected differently by $k_t^{(2)}$. The model assumes that number of deaths follow a Poisson distribution.

The logit transformation of the original CBD model posed by Cairns et al. (2006) is formulated as:

$$q_{x,t} = \frac{e^{k_t^{(1)} + k_t^{(2)}(x - \bar{x})}}{1 + e^{k_t^{(1)} + k_t^{(2)}(x - \bar{x})}}$$

Actually, the formula of the CBD model has been derived from the above transformation.

The fitted parameters $k_t^{(1)}$ and $k_t^{(2)}$ are obtained by applying least squares to the logit transformation of CBD shown above.

There are several approaches attempted in literature to model the time-trend indexes of CBD. Following the steps of Cairns et al. (2006), we shall apply multivariate random walk with drift to forecast $k_t^{(1)}$ and $k_t^{(2)}$. Haberman and Renshaw (2011) and Lovasz (2011) also applied a multivariate random walk with drift to forecast the two parameters of CBD. Indeed, a multivariate random walk with drift is the standard approach to forecasting CBD parameters in actuarial literature. The formula for the multivariate random walk with drift used is:

$$k_t = \delta + k_{t-1} + \xi_t^k,$$

where

$$k = \begin{pmatrix} k_t^{(1)} \\ \vdots \\ k_t^{(N)} \end{pmatrix} \text{ and } \xi_t^k \sim N(0, \Sigma).$$

The parameter δ is an N -dimension vector of drift parameters and Σ is the $N \times N$ variance-covariance matrix of the multivariate white noise ξ_t^k .

Another alternative to model the period effects for forecasting though is a general univariate ARIMA model. In addition, CBD mortality indexes can also be modelled using VARIMA, which was attempted by Chan, Li and Li (2014). They argued that the association that exists between $k_t^{(1)}$ and $k_t^{(2)}$ affects the longevity risk of a portfolio.

Furthermore, if two-dimensional random walk with drift is used to forecast CBD parameters, then its formula becomes:

$$\vec{k}_t = \vec{k}_{t-1} + \mu + C \cdot N(t+1)$$

Whereby,

- $\vec{k}_t$ represents the parameters vector, $\vec{k}_t = [k_t^{(1)}, k_t^{(2)}]$,
- μ represents the arithmetic mean of the differenced series of estimated parameters,
- C has been derived from the Cholesky decomposition of the variance-covariance matrix of the $\vec{k}_t$
- $N(t+1)$ is a two-dimensional standard normal random variable

These forecasted mortality indexes shall then be plugged into the original CBD formulation to obtain the forecasted mortality rates for the data under consideration.

3. MORTALITY DATA

The two main datasets used in this paper are the number of deaths of males and their respective exposure-to risk, which have been both retrieved from the online Human Mortality Database (HMD). Each original dataset provides data for single ages 0-110+, whereby 0 represent babies below 1 year old and 110+ represent individuals who are 110 and older.

3.1 Impact of the economic development of a country on mortality

It is widely believed that economic conditions of a country have a significant impact on mortality modelling. The aim of this study is to analyse the mortality characteristics of different countries in order to better project mortality for different ages. Most of the countries in the world are classified into 3 broad categories, as per their level of development. They are developed, developing and under-developed categories.

Statistical indexes such as income per capita, GDP per capita, life expectancy, literacy rate among others measure the development level of a mortality. There exists a linear relationship between the development level of a country, its economy and advances in its technological infrastructure. This simply means that the more developed a country is, the more sophisticated and higher is its economy and the better quality of life they provide. Indeed, in under-developed countries, citizens have limited access to medical facilities due to inadequate infrastructure and resources. In addition, there is little medical and technological advancements in these countries.

The insurance products in developing and developed countries are different from each other in several ways. In addition, health care system also varies in accordance with the development level of a country. Medical system in developed countries rely mostly on conventional medicine, unlike the developing ones which have a naturopathic system. We therefore conclude that valuation of derivatives which may rely on mortality features, may not be the same for developed and developing countries. With all these discoveries made in medicine, mortality observed in developed countries will certainly be lower than in developing countries, which in turn will be lower than under-developed countries, despite the fact that the lifestyle (drugs, accidents, suicidal tendencies) observed in developed countries can contribute to an unexpected rise in mortality.

3.2 Mortality data selection

For the purpose of this research, we shall employ the mortality data for five countries. The countries are selected taking into consideration their level of development as per the World Economic Situation and Prospects (WESP) report and their geographical location. For instance, France, Japan and Canada are selected as being developed countries while Ukraine and Taiwan being developing countries. Each developed and developing country which has been chosen comes from a specific geographical region, namely Europe, East-Asia and lastly North-America. It is to be noted that no data of a developing country was found from the latter region. Besides, data for under-developed countries across the world are not available in the HMD.

Data which seemed to be unreliable due to age-heaping and poor-quality problems shall be excluded from this analysis. We therefore exclude all mortality data before year 1900 and mortality data for age 90 and above. This is to avoid any specious results which may cause inaccurate mortality forecasts.

For all the countries, the datasets will be split into 2 periods which will be used for an in-sample and out-sample analysis. This is presented in Table 1. The models will be built using the first 70% of the

observations for a reasonable data size when fitting, while the remaining 30% will be kept aside for an out-of-sample analysis.

Table 1. Time horizon of each datasets

Country	In-sample dataset	Out- sample dataset
France	1900-1980	1981-2016
Ukraine	1970-2000	2001-2013
Japan	1947-1996	1997-2016
Taiwan	1985-2007	2008-2014
Canada	1921-1990	1991-2016

We also aim to model mortality for different age groups. We therefore split the mortality data for ages from 0 to 90 years old into 3 specific groups namely:

- i. Group A includes ages 0 to 4 inclusive
- ii. Group B includes ages 5 to 60 inclusive
- iii. Group C includes ages above 60

3.3 Preliminary data analysis

We performed a preliminary analysis on the five mortality datasets. Mortality is experienced differently in different stages of life. For instance, infants initially experience a high mortality rates, which lightly decrease until childhood due to consistent supervision. From adolescence, mortality increases again until by late teenage where accident humps are observed as a result of active lifestyle whereby teenagers get involved in accidents, drug related issues and at this stage, many are vulnerable to educational and social breakdowns leading to suicidal tendencies. Lastly, by the mid age around 40 to 70 years, a steady increase in mortality is observed as individuals get older and health issues start developing, and hence mortality rate increases. We also observed that accidents humps of males over the ages 15-20 are more prominent in developed countries compared to developing countries.

4. APPLICATION OF THE MORTALITY MODELS

We analyse the model fitting of the LC, APC and CBD mortality models for the three age groups (0-4, 5-60 and 60+) from the five populations (France, Japan, Canada, Ukraine and Taiwan). Table 2 describes the pattern of the parameters which are estimated for the mortality models. The model parameters are as follows:

- ${}^{LC}\alpha_x$ and ${}^{APC}\alpha_x$ denote mean pattern of mortality for age x in LC and APC models respectively,
- ${}^{LC}\beta_x^{(1)}$ is the rate of change in the central rate of mortality with respect to changes in ${}^{LC}k_t^{(1)}$ in LC model,

- $^{LC}k_t^{(1)}$, $^{APC}k_t^{(1)}$ and $^{CBD}k_t^{(1)}$ represent the mortality index in LC, APC and CBD models respectively. The mortality index exhibits the period effect which is the relationship between time-dependent events and mortality rates,
- $^{APC}\gamma_{t-x}$ relates to the general level of mortality of the APC model for cohort born in the year $t - x$,
- $^{CBD}k_t^{(2)}$ is the slope of the CBD model which represents improvements in mortality.

The analysis from Table 2 depicts the following salient features in terms of the:

- **Age groups**

There are different mortality patterns which exist for the three age groups. For instance, mortality for Group A (0-4) shows a decreasing trend while mortality for Group B (5-60) exhibits an increasing trend. Mortality increases heavily for Group C (60+).

- **Cohort effect**

The cohort effect is significant for all three age groups from all five countries. This is displayed by the cohort parameter $^{APC}\gamma_{t-x}$ in the APC model.

- **Developing and developed countries**

The cohort effect for developed countries is observed earlier than for developing countries. For example, the mortality declines for cohort born after 1950 for developed countries whereas the mortality declines only after 1975 for the developing countries. This may be explained by the fact that technological and medical advances were slower in developing countries than for developed countries. Another interesting observation is that mortality improvement in age Group C (60+) is negative for the developing countries while it is generally positive for the developed countries.

- **Country-specific factors**

For France and Ukraine, mortality jumps can be observed at some specific years. For example, France mortality display some peaks around year 1920 and year 1940, which correspond to epidemics and World War (I & II) events. For Ukraine, there is a mortality crisis in 1990 after its independence from the Soviet Union.

The observations provide evidence of the distinct mortality pattern which exists in different age groups from countries with different economic levels.

Table 2:

Analysis of the pattern of the model parameters from LC, APC and CBD for the three age groups from the five countries.

	$LC \alpha_x$ & $APC \alpha_x$	$LC \beta_x^{(1)}$	$LC k_t^{(1)}$, $APC k_t^{(1)}$ & $CBD k_t^{(1)}$	$APC \gamma_{t-x}$	$CBD k_t^{(2)}$
Group A (0-4)	- For all 5 countries, a very high infant mortality rate due to congenital disorders, followed by light decrease as the baby grows. Mortality rate is the highest and lowest at age 0 and 4, respectively.	- For France, Canada and Ukraine, it increases from age 0 to 1 and decreases beyond age 1 - For Japan, it increases from age 0 to 2 and decreases after. - For Taiwan, it increases from age 0 to 3 and decreases after.	- For Japan, Canada and Taiwan, a gradual decrease is observed. - For France, some peaks around year 1920 and year 1940 are observed. They correspond to "mortality jumps" which are repercussions of historical events and epidemics. From 1950 onwards, gradual decrease in mortality index is observed, which is mainly ascribed to technological advances and medical discoveries. - For Ukraine, mortality jumps can be noticed around year 1990 which correspond to the mortality crisis in Ukraine after its independence from the Soviet Union.	- Cohort effects are observed for all 5 countries. - In France, mortality was decreasing for males born before in early 20th century and increasing for males born in the late 20th century. This was basically the post-war effects which affected several developed countries. - In Japan, after 1970s, a relatively stable mortality was observed, while in Canada, generations born as from the 1970s benefitted from mortality improvements. - For Ukraine, mortality increases for babies born before 1986 and decreases for babies born after 1986. - For Taiwan, mortality decreases for males born before 1994 and rises for those born after 1994.	- For the developed countries, improvements in infant mortality rates became apparent after 1960s due to an enhanced development in the medical health care sector, despite that some fluctuations are observed beyond 1960s. - For Ukraine, infant mortality was enhancing as from 1975. - For Taiwan, there is a decreasing trend which indicates that there were negative improvements in mortality.
Group B (5-60)	- For all 5 countries, a decline in mortality from age 5 to 16, followed by a temporary increase "accident hump" at late teenage from age 17 to 20, eventually	- For all countries except Ukraine, an increase for ages 17-20, and a decrease for ages 22+. French teenagers have experienced	- For Japan, Canada and Taiwan, a gradual decrease is observed. - For France, there is a general decrease over time except that up-jumps are observed in 1914-1918 and 1939-1945 periods which correspond to the World War I (WWI) and World War II (WWII) respectively.	- Mortality was almost steady for French and Canadian males born in 1900-1950 period while declining for those born in the 1950-2000 period. - Japanese mortality is seen to rise for males born before 1940 and fall for those born after year 1940. - For Ukraine and Taiwan, mortality is seen to rise for males born before	- Progresses in mortality were observed in the mortality rates of Japan, Canada and Taiwan. - For France, mortality rates improved overall except at WWI and WWII where an abrupt rise in mortality is observed.

	increasing steadily till age 60. The hump is less pronounced for developed countries.	mortality at a much faster pace. - For Ukraine, it is seen to increase almost linearly from ages 5 to 20, until it increases decreasingly thereafter.	- For Ukraine, a rise in mortality is seen across time, whereby sporadic demographic losses were observed due to high death rate and a very low birth rate.	1975 and fall for those born after year 1975.	- For Ukraine, mortality rates improved overall except at 1990 where an abrupt rise in mortality is observed.
Group C (60+)	- For all 5 countries, a linear increase as from age 61 till senescence, which implies that mortality increases heavily as from mid age to old age.	- For the 5 countries, different ages showed fluctuations in mortality over the years. This indicates the high volatility of mortality at high ages.	- For Japan, Canada and Taiwan, a gradual decrease is observed. - For France and Ukraine, the same observations as for Groups A & B are made.	- For the 5 countries, a general decrease is observed. However, the level of mortality is much more volatile across the different cohorts than for Groups A & B.	- For the developed countries, progresses were observed in the mortality rates. For France, no significant improvement in the mortality for old ages were seen before 1970s. - For the developing countries, mortality improvement decreases. A slight improvement is observed as from 2000 for Taiwan.

4.1 Performance Metrics

When the mortality rates for each age group for each country are forecasted, we proceed to the forecasting evaluation which is the out-sample testing. It is also referred to as backtesting. It is essential to assess the forecasting power of the 3 stochastic mortality models, by comparing the fitted mortality rates to the actual mortality rates, through the forecast accuracy metrics. The same accuracy metrics in Alijean and Narsoo (2018) are employed. They are the Root Mean Squared Error (RMSE), the Mean Absolute Error (MAE), the Mean Percentage Error (MPE) and the Mean Absolute Percentage Error (MAPE).

The RMSE considers the square of the deviations between actual and forecasted values while the MAE computes the absolute value of those deviations. The MPE and the MAPE both rely on percentage errors and are therefore scale-independent according to Hyndman and Koehler (2006). Each accuracy metric is averaged across the number of forecasted years for each country in order to compute a single metric.

5. RESULTS

The forecasting performance of LC, APC and CBD models are assessed on the basis of the RMSE, MAE, MPE and MAPE metrics for each age category: Group A (0-4), Group B (5-60) and Group C (60+).

5.1 Age Group A (0-4)

Tables 3 to 7 display the single accuracy metrics of mortality rates for French, Japanese, Canadian, Ukrainian and Taiwanese males for the age group 0-4.

Table 3. France - Age group A (0-4) for the years 1900-2016

Models	Average RMSE	Average MAE	Average MPE	Average MAPE
LC	0.0007165635	0.0005016446	31.36798	36.64059
APC	0.0012928940	0.0008281980	-2.919364	44.07071
CBD	0.0010564450	0.0008224977	-18.37344	81.64094

Table 4. Japan - Age group A (0-4) for the years 1947-2016

Models	Average RMSE	Average MAE	Average MPE	Average MAPE
LC	0.00037076400	0.0002467675	16.16976	23.95516
APC	0.00005013275	0.0000444328	-1.450511	8.527995
CBD	0.00040679810	0.0003161512	-7.369442	50.14884

Table 5. Canada - Age group A (0-4) for the years 1921-2016

Models	Average RMSE	Average MAE	Average MPE	Average MAPE
LC	0.0004168549	0.0002490608	12.29496	17.49456

APC	0.0002196822	0.0001489193	-1.848184	15.11301
CBD	0.0006177664	0.0004440933	-22.78797	80.39649

Table 6. Ukraine - Age group A (0-4) for the years 1970-2013

Models	Average RMSE	Average MAE	Average MPE	Average MAPE
LC	0.0002358218	0.0001242271	-0.6975225	3.224312
APC	0.0002587505	0.0002249510	3.77745	14.46273
CBD	0.0012309520	0.0009490906	-6.581539	59.06396

Table 7. Taiwan - Age group A (0-4) for the years 1985-2014

Models	Average RMSE	Average MAE	Average MPE	Average MAPE
LC	0.0004936753	0.0002930085	14.91240	23.13983
APC	0.0010699270	0.0006053886	-6.568397	40.21809
CBD	0.0004560668	0.0003098326	-3.956334	67.61062

For France, the Lee-Carter model gives the smallest value for all the mean forecast metrics, while CBD gives the largest value for the accuracy metrics. The large value of the average MAPE implies that LC is not the finest model to forecast French infant mortality. This is so because Lee-Carter actually fails to capture jumps in mortality which resulted from the two World Wars and the influenza outbreak. An extension to LC model is considered in Alijean and Narsoo (2018) where the LC specification is augmented with the Double Exponential Jump Diffusion model in order to capture the mortality jumps. The augmented model turns out to give the best forecasts for French mortality data. For Japan and Canada, APC gives the lowest accuracy metrics, while CBD displays the worst values. For the developing countries Ukraine and Taiwan, the deviation between the actual and the forecasted mortality rates is less significant for Lee-Carter. Most of the accuracy metrics are found to be least for the Lee-Carter model. The CBD is the least accurate model for the age group A.

5.2 Age Group B (5-60)

Tables 8 to 12 display the single accuracy metrics of mortality rates for French, Japanese, Canadian, Ukrainian and Taiwanese males for the age group 5-60.

Table 8. France - Age group B (5-60) for the years 1900-2016

Models	Average RMSE	Average MAE	Average MPE	Average MAPE
LC	0.0012413000	0.0008698933	-20.329100	36.27018
APC	0.0009218800	0.0006419885	-0.5541069	17.42154
CBD	0.0006183309	0.0005237444	-8.1678870	24.04761

Table 9. Japan - Age group B (5-60) for the years 1900-2016

Models	Average RMSE	Average MAE	Average MPE	Average MAPE
LC	0.0001671769	0.000116521	3.274166	11.52526
APC	0.0002444464	0.0001579079	6.519216	10.44033
CBD	0.0002907615	0.000217814	1.451052	19.74819

Table 10. Canada - Age group B (5-60) for the years 1921-2016

Models	Average RMSE	Average MAE	Average MPE	Average MAPE
LC	0.0009547818	0.0005473839	-0.071863	16.65279
APC	0.0003711195	0.0002297892	6.545201	13.9587
CBD	0.0006524981	0.0004582155	-6.295812	26.69022

Table 11. Ukraine - Age group B (5-60) for the years 1970-2013

Models	Average RMSE	Average MAE	Average MPE	Average MAPE
LC	0.0008555914	0.0004282105	-3.558388	5.923989
APC	0.0012095430	0.0005825007	1.705617	5.878216
CBD	0.0012113040	0.0007149815	-21.10156	28.60067

Table 12. Taiwan - Age group B (5-60) for the years 1985-2014

Models	Average RMSE	Average MAE	Average MPE	Average MAPE
LC	0.0001750134	0.0001240401	-0.680572	7.488498
APC	0.0001723755	0.000120841	-1.446688	8.29718
CBD	0.000275859	0.0002033138	-18.60519	24.92991

For France, the overall metrics show that CBD model outperforms APC which in turn outperforms Lee-Carter model. The high accuracy metrics are evidence that the Lee-Carter performs the least accurately for this age group. This significant underestimation is principally because of the several deadly events which resulted in a greater death toll of young French adults who were in the military. On the other hand, the CBD model which is generally more appropriate for volatile mortality rates at older ages proves to capture well the mortality structure of 5-60 French males. For Japan, LC is found to perform the best given its low values of the accuracy metrics, while CBD turns out to be the poorest model. For Canada, APC gives the lowest of all the forecast accuracy metrics while LC on the other hand, displays the largest values of the errors. For the developing countries, Ukraine and Taiwan, LC model exhibits the smallest of most of the accuracy metrics and CBD the largest, which indicates that LC is more accurate than APC and CBD. We also note that APC performs fairly for both cases. The CBD reveals to be the least accurate model for the age group B.

5.3 Age Group C (60+)

Tables 13 to 17 display the single accuracy metrics of mortality rates for French, Japanese, Canadian, Ukrainian and Taiwanese males for the age group 60+.

Table 13. France - Age group C (60+) for the years 1900-2016

Models	Average RMSE	Average MAE	Average MPE	Average MAPE
LC	0.005021458	0.003344792	0.4846488	3.143864
APC	0.008229611	0.004472263	1.342061	3.059214
CBD	0.006669327	0.004146146	1.811669	3.892694

Table 14. Japan - Age group C (60+) for the years 1947-2016

Models	Average RMSE	Average MAE	Average MPE	Average MAPE
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LC	0.002445428	0.001864717	0.0151552	5.535133
APC	0.0009040844	0.0005882695	-0.4859698	1.795141
CBD	0.001751536	0.001112245	0.03123846	2.914073

Table 15. Canada - Age group C (60+) for the years 1921-2016

Models	Average RMSE	Average MAE	Average MPE	Average MAPE
LC	0.00825114	0.005070381	-0.1932626	5.886142
APC	0.005244855	0.003054745	0.1254501	3.45739
CBD	0.004413218	0.002460389	0.0860601	2.640085

Table 16. Ukraine - Age group C (60+) for the years 1970-2013

Models	Average RMSE	Average MAE	Average MPE	Average MAPE
LC	0.007815107	0.005443768	-3.477271	3.884897
APC	0.003662685	0.003014674	-2.347882	3.260519
CBD	0.01222527	0.007804058	-0.9866851	5.878508

Table 17. Taiwan - Age group C (60+) for the years 1985-2014

Models	Average RMSE	Average MAE	Average MPE	Average MAPE
LC	0.001503298	0.001142224	-0.8247774	4.454157
APC	0.001038697	0.0007486365	-0.1840023	2.536097
CBD	0.001176554	0.0009248405	0.5482153	2.866132

For the ages above 60, it can be observed that the accuracy metrics for all models are small enough, that is, all 3 models are able to project old age mortality in all countries. Furthermore, the difference observed between the best and worst model is insignificant, which in a way makes it more difficult to judge the performance of the models for the males.

For France, given that the accuracy metrics are the smallest for LC implies that the latter model is better able to forecast mortality rates while APC turns out to poorly estimate higher ages mortality. For Canada, CBD outperforms APC and LC as its degree of wrong estimation is the smallest with respect to the remaining models. In Japan, Ukraine and Taiwan, APC is found to outperform LC and CBD.

As observed above, APC and CBD model mortality for higher ages perform really well for all the countries regardless of their development scales. There exists a growing amount of literature on the application of CBD model on mortality modelling for higher ages. We therefore further our analysis by implementing CBD model on a 75+ age group for the five countries.

5.4 Age Group 75+

There exists a growing amount of literature on the application of CBD model on mortality modelling for higher ages. We therefore further our analysis by implementing CBD model on a 75+ age group for the five countries. Tables 18 to 22 display the single accuracy metrics of mortality rates for French, Japanese, Canadian, Ukrainian and Taiwanese males for the age group 75+.

Table 18. France - Age group 75+ for the years 1900-2016

Models	Average RMSE	Average MAE	Average MPE	Average MAPE
LC	0.004916253	0.003690754	1.867295	2.31088
APC	0.006847519	0.004962387	2.381423	2.801811
CBD	0.005282755	0.003815062	2.001622	2.169866

Table 19. Japan - Age group 75+ for the years 1947-2016

Models	Average RMSE	Average MAE	Average MPE	Average MAPE
LC	0.0006551329	0.0005949433	0.5427624	1.171324
APC	0.001219359	0.0009911133	1.474119	1.722773
CBD	0.000710212	0.0005806933	0.6360009	0.9735336

Table 20. Canada - Age group 75+ for the years 1921-2016

Models	Average RMSE	Average MAE	Average MPE	Average MAPE
LC	0.007730471	0.005730633	1.489382	4.233619
APC	0.004296009	0.00276761	1.361738	2.139309
CBD	0.004062484	0.002595713	0.8583085	1.873035

Table 21. Ukraine - Age group 75+ for the years 1970-2013

Models	Average RMSE	Average MAE	Average MPE	Average MAPE
LC	0.007244745	0.005886647	-3.142683	3.303514
APC	0.004795516	0.004179579	-2.601312	2.770804
CBD	0.006004446	0.004746706	-2.616266	3.005467

Table 22. Taiwan - Age group 75+ for the years 1985-2014

Models	Average RMSE	Average MAE	Average MPE	Average MAPE
LC	0.00165829	0.00101626	1.169329	2.214367
APC	0.001596859	0.001398067	0.4216047	2.593112
CBD	0.001512875	0.001194487	1.134187	2.014059

For France, Japan and Taiwan, the accuracy metrics are very small for both LC and CBD, which implies that both models are performant to forecast mortality rates of French males in the 75+ age group. APC is the least performant. For Canada, CBD clearly outperforms APC and LC as per its lowest accuracy metrics. For Ukraine, APC is found to perform best while LC is the least accurate.

On the basis of the MAPE metrics, we observe that CBD is the most accurate model for France, Japan, Canada and Taiwan.

6. CONCLUSION

This study evaluated the forecasting accuracy of the Lee-Carter (LC), Age-Period-Cohort (APC) and Cairns-Blake-Dowd (CBD) mortality models for three age groups from five countries. Mortality data for three developed countries (France, Japan and Canada) and two developing countries (Ukraine and Taiwan) are considered for age groups A (0 to 4), B (5 to 60) and C (above 60). The forecasting performance is assessed on the basis of RMSE, MAE, MPE and MAPE metrics. LC was found to be more accurate than its extensions for both developed and developing for the age groups A and B. CBD

was significantly the poorest model in modelling mortality for males at ages below 60 except for France age group B. This particular group suffered several deadly events which resulted in a greater death toll of young French adults who were in the military. The CBD model which generally performs well for highly volatile mortality rates at older ages proves to capture well the mortality structure of 5-60 French males. APC performed best for the age group C. In a further analysis, CBD proved to provide best mortality projections for the 75+ age group.

It is observed that the mortality for developing countries for each age group is best modelled by the same model: LC for 0-4 and 5-60 age groups, APC for 60+ and CBD for 75+. This provides evidence of the similar mortality pattern which exists for developing countries. However, the best performing mortality model is not same across the three developed countries for each age group. This may be explained by the different historical events which have occurred in each of these countries.

In a future research, we intend to explore extensions of the models which may better capture mortality patterns, thus providing more precise forecasts. Also, this analysis may be further extended by considering a larger dataset with more developed and developing countries and including under-developed countries. However, limited accessibility of data for under-developed countries may be a major obstacle.

7. REFERENCES

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