

Influence of fear in a ratio-dependent predator-prey model

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ABSTRACT

Modelling of predation process of an eco-system becomes closer to the existent situation when response of both the species is believed in the modelling process. Ratio-dependent functional response is one of them. In this article, a prey-predator model is thought with fear effect for prey and ratio-dependent Holling type II functional response predation development. The crucial mathematical features of the proposed model are analyzed with the help of equilibria, stability analysis, and bifurcation theory. The parametric space under which the system enters into a Hopf-bifurcation has been investigated. Explicit formula for determining the stability of bifurcating periodic solutions is driven by using normal form and central manifold theory. Our analytical findings are performed by numerical experiments. Biological implications of the analytical findings are talked about in the conclusion section.

Keywords: Prey-predator model; Ratio-dependent response; Fear effect; Stability; Hopf-bifurcation.

Mathematics Subject Classification: 92Bxx, 92D40, 37B25, 34D23, 34D35

Journal of Economic Literature (JEL) Classification : C63, P28, Q57

1. INTRODUCTION

Predation mechanism has the key role of research in theoretical ecology from the early beginning of this discipline [see Abrams, P.A. (1994), Freedman, H. (1980), Ko, W. & Ryu, K. (2006), Haque, M., Rahaman, M. S., Venturino, E. & Li, B.-L. (2014)]. Studies of mathematical models are informative in understanding prey-predator interactions in these systems; as a result, predator-prey dynamics have been significant in mathematical ecology since the early days of this discipline after the pioneer work of Lotka and Volterra. We begin with the modified Lotka-Volterra type prey-predator system which is known as Wangersky-Cunningham model

$$\frac{dx}{dt} = rx - d_1x - h_1x^2 - mxy, \quad (1.1a)$$

$$\frac{dy}{dt} = emxy - d_2y, \quad (1.1b)$$

$$x(0) \geq 0, y(0) \geq 0, \quad (1.1c)$$

where $x(t)$ and $y(t)$ are the population densities of prey and predator respectively; r is the birth rate, d_1 is the death rate of prey ($r > d_1$), h_1 is the intra-specific competition among prey populations e ($0 < e < 1$) is the conversion factor, m is the predation rate of predator and d_2 is the death rate of predator. In 2009, (Haque, 2009) studied the system (1.1) by incorporating ratio-dependent functional response and intra-specific competition among predator populations.

In 2016, (Wang, Zanette & Zou, 2016) first incorporate fear effect predator-prey model (1.1) and conclude that high level of fear can stabilize the system by excluding the existence of periodic solutions. But most of the researchers develop prey-dependent predator-prey model considering fear effect proposed by Wang now [cf. (Zhang, H., Cai, Y., Fu, S. & Wang, W. (2019), Panday, P., Samanta, S., Pal, N. & Chattopadhyay, J. (2020), Sarkar, K. & Khajanchi, S. (2020)].

The notion of ratio-dependent functional response in a predator-prey model system can be merely acknowledged as the per capita predator growth rate should be a function of the ratio of the prey to predator abundance. Progressive evidence points out that in many cases where predators must search, share and compete for food, ratio-dependent predator-prey models are considered realistic. An investigation provided in (Hsu, Hwang & Kuang, 2001) demonstrates that the ratio-dependence models are competent of generating comfortable and more realistic dynamics than the models thinking about only prey-dependent functional response. Ratio-dependent predator-prey models can exhibit remarkable and innovative dynamical properties which are not examined in simple two-dimensional predator-prey models.

To put it briefly, the reaction functions have vital outcome on the dynamical behaviour of predator-prey models, the functional response is categorized into four types, that is, Lotka-Volterra type, Holling type II or Michaelis-Menten type, generalized Holling type III or Sigmoidal, Holling type IV functional response. In this investigation, we extend the study of the classical form of the predator prey model (1.1) incorporating the Holling type II ratio-dependent functional response. One of the aims of this study is to explore a Holling type II ratio-dependent predator-prey model and the originality of this investigation lies with the inclusion of fear effect for prey species.

The objective of this paper is to analyze predator-prey system with fear effect in prey in a classical predator-prey model. To construct the mathematical model, we believe ratio-dependent response function for predation process. Account for these scenario, one can obtain the strongly coupled nonlinear predator-prey system with fear effect for prey species.

2. The mathematical model

According to the above assumptions, the model (1.1) becomes

$$\frac{dx}{dt} = \frac{rx}{1+ky} - d_1x - h_1x^2 - \frac{mxy}{ax+y}, \quad (2.1a)$$

$$\frac{dy}{dt} = \frac{emxy}{ax+y} - d_2y - h_2y^2, \quad (2.1b)$$

$$x(0) \geq 0, y(0) \geq 0, \quad (2.1c)$$

where the system parameters $r, k, d_1, h_1, m, a, e, d_2, h_2$ are all positive constants; $r, k, d_1, h_1, m, e, d_2$ are defined earlier in model (1.1); a is the saturation constant, h_2 is the intra-specific competition among predator populations.

3. Preliminary results

3.1. Existence and positive invariance

For $t > 0$, letting $X \equiv (x, y)^T, F: R^2 \rightarrow R^2, F = (f_1, f_2)^T$, system (4.2) can be rewritten as $\frac{dX}{dt} = F(X)$. Here $f_i \in C^\infty(R)$ for $i = 1, 2$; where $f_1 = \frac{rx}{1+ky} - d_1x - h_1x^2 - \frac{mxy}{ax+y}, f_2 = \frac{emxy}{ax+y} - d_2y - h_2y^2$.

Since the vector function F is a smooth function of the variables (x, y) in the positive quadrant $\Omega = \{(x, y): x > 0, y > 0\}$, the local existence of the solution hold.

3.2. Boundedness results

Boundedness implies that the system is biologically well behaved. The following propositions ensure the boundedness of the system (2.1).

3.2.1. The solution of the system (2.1) which initiate R_+^2 in are uniformly bounded.

Proof. Define a positive definite function

$$\Omega(t) = x(t) + \frac{1}{e}y(t). \quad (3.1)$$

From definition, $\Omega(t)$ is differentiable in some maximal interval $(0, t_b)$. For any arbitrary $\eta > 0$, the time derivative of (3.1) along the solution of the system (2.1) is

$$\begin{aligned} \frac{d\Omega}{dt} + \eta\Omega &= x\left(\eta + \frac{r}{1+ky} - d_1 - h_1x\right) + y\left(\eta - \frac{d_2}{e} - \frac{h_2}{e}y\right) \\ &\leq x(\eta + r - d_1 - h_1x) + y\left(\eta - \frac{d_2}{e} - \frac{h_2}{e}y\right) \\ &\leq \frac{(\eta + r - d_1)^2}{4h_1} + \frac{\left(\eta - \frac{d_2}{e}\right)^2}{4\frac{h_2}{e}}. \end{aligned}$$

Hence, we can find $\mu > 0$ such that

$$\frac{d\Omega}{dt} + \eta\Omega \leq \mu \quad \forall t \in (0, t_b).$$

Applying the theory of differential equation [Birkhof, G.& Rota, G. C., (1989)], we get

$$0 < \Omega(x, y) < \frac{\mu}{\eta}(1 - e^{-\eta t}) + \Omega(x(0), y(0))e^{-\eta t} \quad \forall t \in (0, t_b).$$

Hence, all the solutions of the system (2.1) that initiate at $(x(0), y(0))$ lie in R_+^2 and are confined in the compact region

$$\Gamma = \left\{(x, y) \in R_+^2; x(t) + \frac{1}{e}y(t) = \frac{\mu}{\eta} + \epsilon \quad \forall t > 0\right\}.$$

3.2.2. If $em > ad_2$, then the system (2.1) is dissipative.

Proof. By standard simple arguments, we obtain from the first equation of the system (2.1) that

$$\lim_{t \rightarrow \infty} \sup x(t) \leq \frac{r-d_1}{h_1} = \bar{x}(\text{say}), \text{ where we denote } \bar{x} \text{ is a unique value of } x(t) \text{ which is positive as } r > d_1.$$

Now, from the second equation of the system (2.1), we observe that

$$\frac{dy}{dt} = -d_2y - h_2y^2 + \frac{\frac{emy}{a}}{1 + \frac{y}{ax}} \leq y \left(\frac{em}{a} - d_2 - h_2y \right).$$

Therefore, $\lim_{t \rightarrow \infty} \text{Sup } y(t) \leq \frac{em - ad_2}{ah_2} = \bar{y}$ (say), where we denote $\bar{y}$ is a unique value of $y(t)$ which is positive as $em > ad_2$. Thus, we conclude that if $em > ad_2$, then the system (2.1) is dissipative.

3.3. If $r - d_1 > m > \frac{ad_2}{e}$, then the system (2.1) is permanent.

Proof. From the first equation of the system (2.1), we have

$$\frac{dx}{dt} = rx - d_1x - h_1x^2 - \frac{mx}{\frac{ax}{y} + 1} \geq x(r - d_1 - m - h_1x).$$

If $\underline{x}$ is the root of $r - d_1 - m - h_1x = 0$, then by standard comparison argument, we observe that the above inequality implies $\lim_{t \rightarrow \infty} \text{Inf } x(t) \geq \underline{x} = \frac{r - d_1 - m}{h_1}$.

From the second equation of the system (2.1), we have

$$\frac{dy}{dt} = -d_2y - h_2y^2 + \frac{emy/a}{1 + \frac{y}{ax}} \geq y \left(-d_2 - h_2y + \frac{emx}{ax + y} \right).$$

If $\underline{y}$ is the positive root of the quadratic equation $-d_2 - h_2y + \frac{emx}{ax + y} = 0$, then $y = \frac{-(ah_2\underline{x} + d_2) + \sqrt{(ah_2\underline{x} + d_2)^2 - 4h_2(ad_2\underline{x} - em\underline{x})}}{2h_2}$. Therefore, $\lim_{t \rightarrow \infty} \text{Inf } y(t) \geq \underline{y}$.

Note that $\underline{x} > 0$ if $r - d_1 > m$ and $\underline{y} > 0$ if $m > \frac{ad_2}{e}$. Therefore, combining these along with dissipative property yields the conclusion that the system (2.1) is permanent if $r - d_1 > m > \frac{ad_2}{e}$.

3.4. Equilibria and their feasibility

System (2.1) has the following three positive equilibria $(2.1)E_i(x_i, y_i), i = 1, 2, 3$. E_0 is the origin, $E_1 \equiv (x_1, 0)$, $E_2 \equiv (x_2, y_2)$, where $x_1 = \frac{r - d_1}{h_1}$ and x_2, y_2 are positive root of the equations

$$\begin{aligned} \frac{r}{1 + ky} - d_1 - h_1x - \frac{my}{ax + y} &= 0 \\ \frac{emx}{ax + y} - d_2 - h_2y &= 0 \end{aligned}$$

4. Stability analysis

The Jacobian matrix of the system (2.1) at any arbitrary point is given by

$$J(x, y) = \begin{bmatrix} \frac{rk}{1 + ky} - d_1 - 2h_1x - \frac{my}{ax + y} - \frac{maxy}{(ax + y)^2} & -\frac{r kx}{(1 + ky)^2} - \frac{max^2}{(ax + y)^2} \\ \frac{emy^2}{(ax + y)^2} & \frac{emx}{ax + y} - \frac{emxy}{(ax + y)^2} - d_2 - 2h_2y \end{bmatrix}. \quad (4.1)$$

4.1. System behaviour near the origin

Since the ratio-dependent model always has difficult dynamics in the neighbourhood of the origin and system (2.1) thus cannot be linearized there; we agree to the technique of Kuang and Beretta (Kuang & Beretta, 1998) with Xiao and Ruan (Xiao & Ruan, 2001) similar to $\frac{dx}{dt} = \frac{dy}{dt} = 0$, when $x = y = 0$.

4.2. System behaviour near the equilibrium E_1

(i) E_1 is locally asymptotically stable if $em < ad_2$.

(ii) Local stability of E_1 ensures its global stability if $\frac{mx_1}{a} + \frac{em}{a} < d_2$.

Proof: (i) The Jacobian matrix at the equilibrium E_1 is $J_1 = \begin{bmatrix} -r + d_1 & -\frac{rk(r-d_1)}{h_1} - \frac{m}{a} \\ 0 & \frac{em}{a} - d_2 \end{bmatrix}$.

The eigenvalues of the Jacobian matrix J_1 at E_1 are $-r + d_1 < 0$, $\frac{em}{a} - d_2$. Hence E_1 is locally asymptotically stable if the condition $em < ad_2$ is satisfied.

(ii) Let $R^2 = \{(x, y) \in R^2_+, x > 0, y \geq 0\}$ and consider the scalar function $L_1: R^2 \rightarrow R$ defined by $L_1 = \frac{1}{2}(x - x_1)^2 + y$. (4.2)

The derivative of the above equation (4.2) along the solution of the system (2.1) is given by

$$\begin{aligned} \frac{dL_1}{dt} &= (x - x_1) \frac{dx}{dt} + \frac{dy}{dt} \\ &= (x - x_1) \left[\frac{rx}{1+ky} - d_1x - h_1x^2 - \frac{mxy}{ax+y} \right] + \left[\frac{emxy}{ax+y} - d_2y - h_2y^2 \right] \\ &\leq -xh_1(x - x_1)^2 - \frac{rkxy}{1+ky} - \frac{mx^2y}{ax+y} + \frac{mxx_1y}{ax+y} + \frac{emxy}{ax+y} - d_2y - h_2y^2 \quad (\text{Since } d_1 = r - h_1x_1) \\ &\leq -xh_1(x - x_1)^2 - \frac{mx^2y}{ax+y} + \frac{mx_1}{a}y + \frac{em}{a}y - d_2y - h_2y^2 \\ &\leq -xh_1(x - x_1)^2 - \frac{mx^2y}{ax+y} + \left(\frac{mx_1}{a} + \frac{em}{a} - d_2 \right) y \\ &< 0 \quad \text{if } \frac{mx_1}{a} + \frac{em}{a} < d_2 \end{aligned} \quad (4.3)$$

and $\frac{dL_1}{dt} = 0$ when $(x, y) = (x_1, 0)$. The proof follows from (4.3) and Lyapunov-Lasale invariance principle [cf. (Hale, 1969)].

4.3. System behaviour near the equilibrium $E_2(x_2, y_2)$

(i) E_2 is locally asymptotically stable if $may_2 < h_1(ax_2 + y_2)^2$.

(ii) The system experiences Hopf-bifurcation around E_2 for $m = m^{[HB]}$, where

$$m^{[HB]} = \frac{(h_1x_2 + h_2y_2)(ax_2 + y_2)^2}{ax_2y_2}.$$

(iii) Local stability of E_2 ensures its global stability if $a < e$.

Proof: (i) The Jacobian matrix J_2 evaluated at E_2 is given by $J_2 = (c_{ij})_{2 \times 2}$, where

$$c_{11} = -h_1x_2 + \frac{max_2y_2}{(ax_2 + y_2)^2}, c_{12} = -\frac{rkx_2}{(1+ky_2)^2} - \frac{amx_2^2}{(ax_2 + y_2)^2} < 0, c_{21} = \frac{emy_2^2}{(ax_2 + y_2)^2} > 0, c_{22} = -h_2y_2 < 0. \text{ Its}$$

$$\text{eigenvalues are } \lambda_{1,2} = \frac{1}{2} \left[c_{11} + c_{22} \pm \sqrt{(c_{11} + c_{22})^2 - 4(c_{11}c_{22} - c_{12}c_{21})} \right]. \quad (4.4)$$

If we assume $c_{11} < 0$, then $\lambda_{1,2}$ both are either negative or complex numbers with negative real parts.

Hence, E_2 is locally asymptotically stable if $c_{11} < 0$, $c_{22} < 0$, that is, $may_2 < h_1(ax_2 + y_2)^2$.

(ii) From (4.4), we see that λ_1 and λ_2 are purely imaginary if and only if there is a $m = m^{[HB]}$ such that

$$m^{[HB]} = \frac{(h_1x_2 + h_2y_2)(ax_2 + y_2)^2}{ax_2y_2}. \text{ But for } i = 1, 2$$

$$Re\left(\frac{d\lambda_i}{dr}\right)_{at [r=m^{[HB]}]} = \frac{ax_2y_2}{(ax_2 + y_2)^2} \neq 0.$$

Therefore, the system enters into Hopf-bifurcation at E_2 for $m = m^{[HB]}$.

$$(iii) \text{ Let } b_1(x, y) = \frac{1}{xy}, x > 0, y > 0 \text{ and } D_1 = \frac{\partial(b_1F_1)}{\partial x} + \frac{\partial(b_1F_2)}{\partial y}.$$

Then we obtain,

$$\begin{aligned} D_1 &= -\frac{1}{xy} \left(h_1x - \frac{maxy}{(ax + y)^2} \right) - \frac{1}{xy} \left(\frac{emxy}{(ax + y)^2} + h_2y \right) \\ &= -\frac{h_1}{y} - \frac{h_2}{x} - \frac{1}{xy} \frac{maxy}{(ax + y)^2} (a - e). \end{aligned}$$

By Dulac criteria, we see that if $a < e$, then $D_1 < 0$; therefore, the system has no non-trivial positive periodic solution. Again, $r - d_1 > m > \frac{ad_2}{e}$ assures that the system is permanent. These together with the assumption of the local stability yield the conclusion.

5. Stability and direction of Hopf-bifurcation

In this section, our concentration is focused on investigation of the stability of the periodic solution bifurcating from a stable equilibrium $E_2(x_2, y_2)$. For this, we consider $u = x - x_2, v = y - y_2$. Then the system (2.1) reduces to

$$\frac{du}{dt} = b_1u + b_2v + \sum b_{ij}u^i v^j,$$

$$\frac{dv}{dt} = c_1u + c_2v + \sum c_{ij}u^i v^j$$

$$i \geq 0, j \geq 0, i + j \geq 2,$$

Where

$$b_1 = \frac{r}{1+ky_2} - d_1 - 2h_1x_2 - \frac{my_2^2}{(ax_2 + y_2)^2}, b_2 = -\frac{rkx_2}{(1+ky_2)^2} - \frac{mx_2^2}{(ax_2 + y_2)^2},$$

$$c_1 = \frac{emy_2^2}{(ax_2 + y_2)^2}, c_2 = \frac{emax_2^2}{(ax_2 + y_2)^2} - d_2 - 2h_2y_2,$$

$$b_{20} = \frac{may_2^2}{(ax_2 + y_2)^3} - h_1, b_{11} = \frac{rk}{(1+ky_2)^2} - \frac{2max_2y_2}{(ax_2 + y_2)^3}, b_{02} = \frac{rk^2x_2}{(1+ky_2)^3} + \frac{max_2^2}{(ax_2 + y_2)^3},$$

$$c_{20} = -\frac{emay_2^2}{(ax_2 + y_2)^3}, c_{11} = \frac{2emax_2y_2}{(ax_2 + y_2)^3}, c_{02} = -\frac{emax_2^2}{(ax_2 + y_2)^3} - h_2,$$

$$b_{30} = -\frac{ma^2y_2^2}{(ax_2 + y_2)^4}, b_{21} = \frac{may_2(2ax_2 - y_2)}{(ax_2 + y_2)^4}, b_{12} = \frac{rk^2}{(1+ky_2)^3} + \frac{max_2(2y_2 - ax_2)}{(ax_2 + y_2)^4}, b_{03} = \frac{rk^3}{(1+ky_2)^4} - \frac{max_2^2}{(ax_2 + y_2)^4},$$

$$c_{30} = \frac{ema^2y_2^2}{(ax_2 + y_2)^4}, c_{21} = \frac{ema(a^2x_2^2 + 2y_2^2)}{(ax_2 + y_2)^4}, c_{12} = -\frac{em(2a^2x_2^2 + y_2^2)}{(ax_2 + y_2)^4}, c_{03} = \frac{emax_2^2}{(ax_2 + y_2)^4}.$$

Following the book of Lawrence Perko (Perko, 2006), one can obtain the Lyapunov number σ as follows:

$$\begin{aligned} \sigma &= -\frac{3\pi}{2b_2\Delta^{3/2}} \left[\{b_1c_1(b_{11}^2 + b_{11}c_{02} + b_{02}c_{11}) + b_1b_2(c_{11}^2 + b_{20}c_{11} + b_{11}c_{02}) + c_1^2(b_{11}b_{02} + 2b_{02}c_{02}) - \right. \\ &2b_1c_1(c_{02}^2 - b_{20}b_{02}) - 2b_1b_2(b_{20}^2 - c_{20}c_{02}) - b_2^2(2b_{20}c_{20} + c_{11}c_{20}) + (b_2c_1 - 2b_1^2)(c_{11}c_{02} - b_{11}b_{20})\} - \\ &\left. (b_1^2 + b_2c_1)\{3(c_1b_{03} - b_2b_{30}) + 2b_1(b_{21} + c_{12}) + (c_1b_{12} - b_2c_{21})\} \right] \quad (5.2) \end{aligned}$$

The bifurcating periodic solutions are stable (unstable) if $\sigma > 0$ ($\sigma < 0$). Hence, Hopf-bifurcation is supercritical (subcritical) if $\sigma > 0$ ($\sigma < 0$).

6. Numerical simulation

In this section, numerical experiments are performed to investigate the dynamics of the system (2.1). Global stability of the system around the equilibrium point E_1 is plotted in Figure 1 for the set of parameter values $r = 1.4, k = 0.5, d_1 = 0.7, m = 1.0, a = 1.0, e = 0.8, d_2 = 3.0, h_1 = 0.5, h_2 = 0.2$ where the stability conditions $em = 1.2 < ad_2 = 3.0$ and $\frac{mx_1}{a} + \frac{em}{a} = 2.2 < d_2 = 3.0$ are satisfied. Figure 2 represents the occurrence of hopf-bifurcation around the equilibrium point E_2 for the set of parameter values $r = 1.5, k = 0.38, d_1 = 0.1, m = 0.75, a = 1.2, e = 0.8, d_2 = 0.1, h_1 = 0.01, h_2 = 0.01$. Figure 3 shows the global stability of the system around the equilibrium point E_2 for the set of parameter values $r = 1.25, k = 0.05, d_1 = 0.3, m = 0.85, a = 0.7, e = 0.8, d_2 = 0.2, h_1 = 0.1, h_2 = 0.1$ where the stability conditions $r - d_1 = 0.95 > m > \frac{ad_2}{e} = 0.175$ and $may_2 = 1.685401048 < h_1(ax_2 + y_2)^2 = 2.578781239$ are satisfied. Impact of fear on the system dynamics are plotted in Figure 4 which indicates that the system changes its behaviour from stable steady state to limit cycle, that is, significant level of fear has the destabilizing potential. Figure 5 & 6 shows that prey and predator population sizes are gradually decreasing with respect to the fear parameter k .

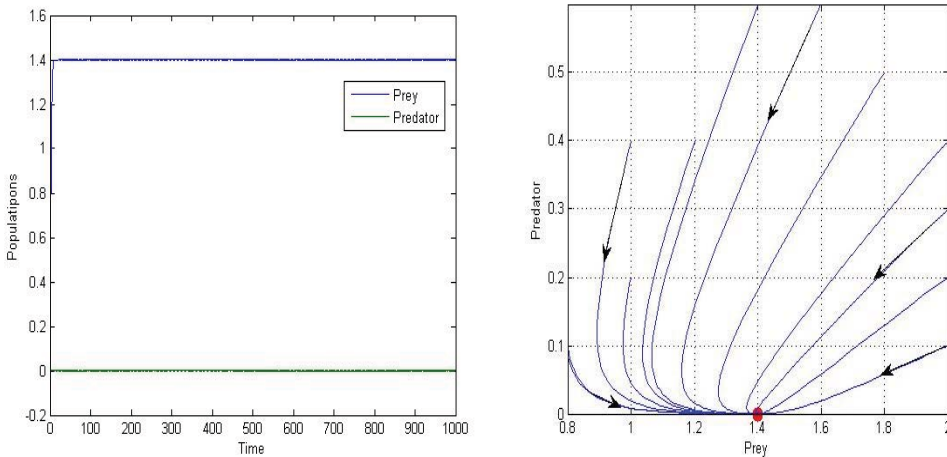


Figure 1: Global stability around axial equilibrium position E_1 of the system (2.1) for the set of parameter values $r = 1.4, k = 0.5, d_1 = 0.7, m = 1.0, a = 1.0, e = 0.8, d_2 = 3.0, h_1 = 0.5, h_2 = 0.2$.

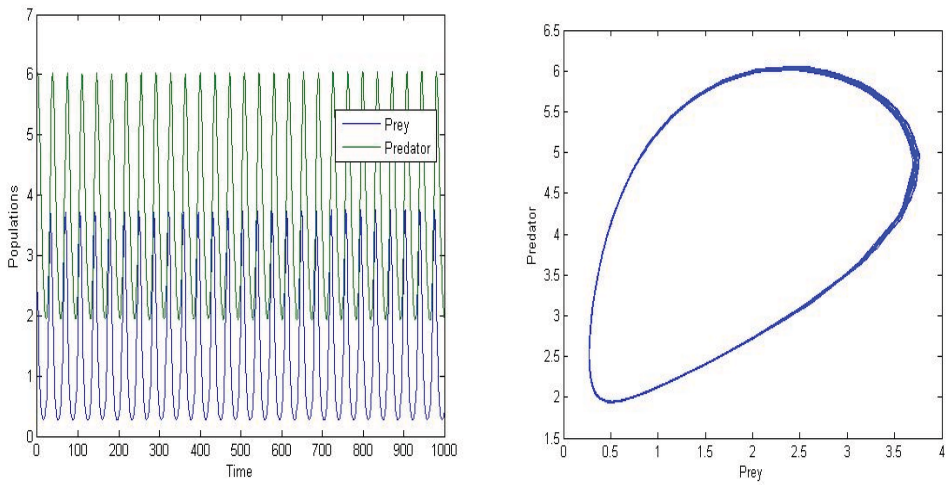


Figure 2: Hopf-bifurcation around interior equilibrium position E_2 of the system (2.1) for the set of parameter values $r = 1.5, k = 0.38, d_1 = 0.1, m = 0.75, a = 1.2, e = 0.8, d_2 = 0.1, h_1 = 0.01, h_2 = 0.01$.

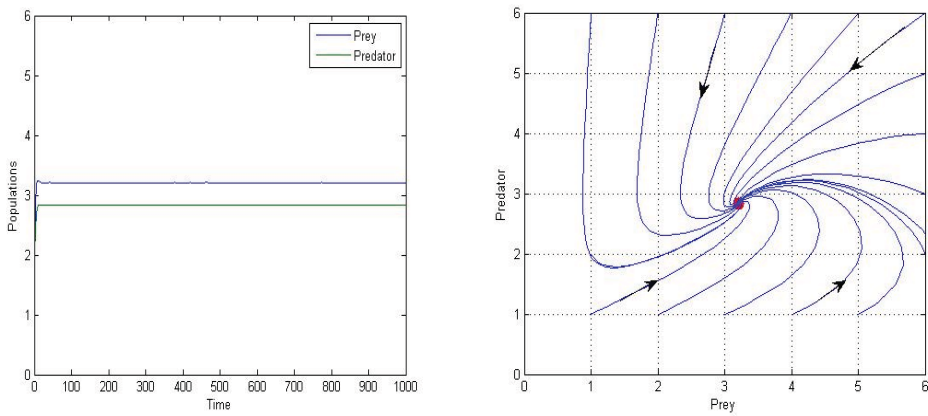


Figure 3: Global stability around interior equilibrium position E_2 of the system (2.1) for the set of parameter values $r = 1.25, k = 0.05, d_1 = 0.3, m = 0.85, a = 0.7, e = 0.8, d_2 = 0.2, h_1 = 0.1, h_2 = 0.1$.

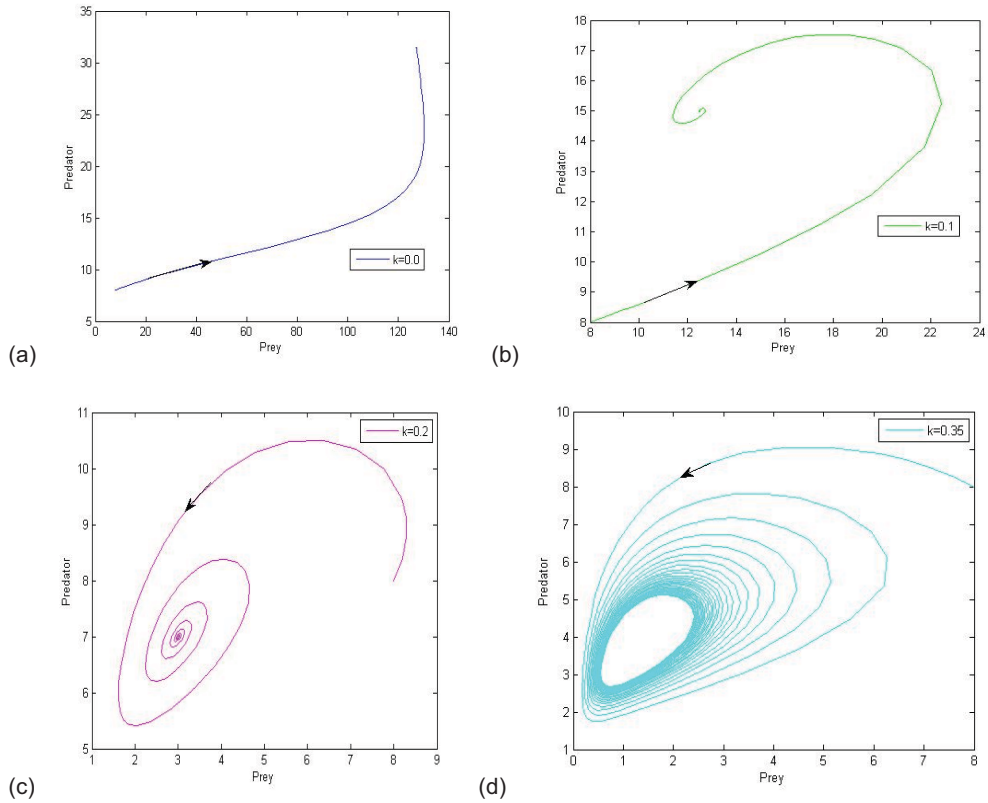


Figure 4: Impact of fear effect on the dynamics of the system (2.1) around interior equilibrium position E_2 . The rest of the parameter values are same as mentioned in Figure 2.

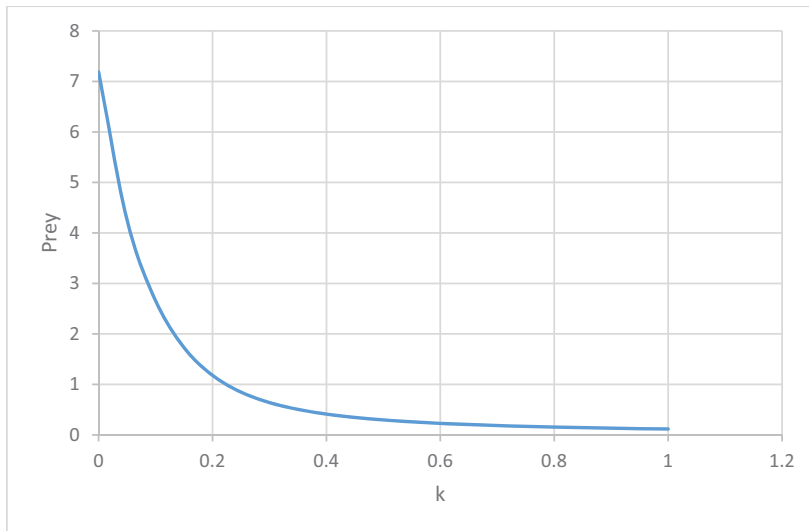


Figure 5: Impact of fear effect on the prey populations of the system (2.1) for the combination of parameter values $r = 1.4$, $d_1 = 0.2$, $m = 1.0$, $a = 0.7$, $e = 0.9$, $d_2 = 0.2$, $h_1 = 0.1$, $h_2 = 0.1$.

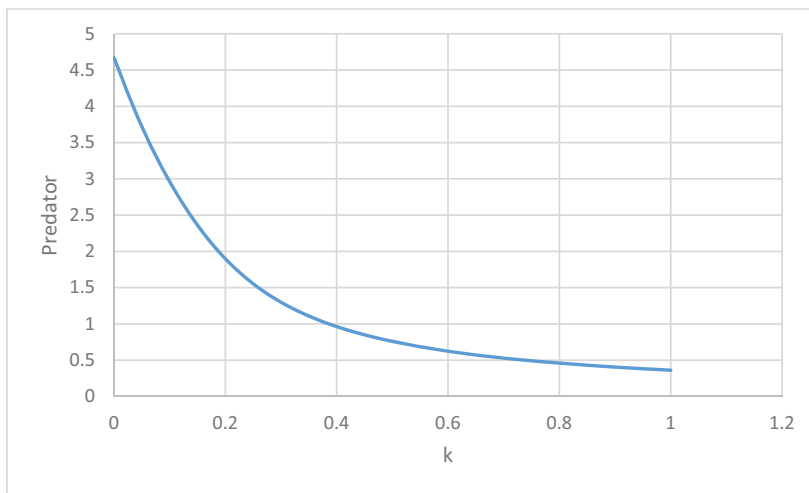


Figure 6: Impact of fear effect on the predator populations of the system (2.1). The rest of the parameter values are same as mentioned in Figure 5.

7. Result and Discussion

We have analyzed the system mathematically and illustrated some of its biological applications as well. A noteworthy numeral of attempts have been made to analyze and to understand the entire mechanisms and their operators of the model (2.1) in the last few decades [cf. (Berezovskaya, Karev & Arditi, 2001) - (Haque, Ali & Chakravarty, 2013)]. Several vital elements in the model (2.1) are the per capita rate of consumption of prey by predator species, cost of fear and death rate of predators which are interpreted as behavioural features of the predator species. It has driven out eventually that the system can show diverse dynamical behaviours such as steady state, bifurcating oscillations etc. as well depending on the values of system parameters. In this paper, we have scrutinized the stability and bifurcation properties of a modified Bazykin's model with fear effect for prey species and ratio-dependent functional response for predation process. Even though bifurcations in a predator-prey model system with Verhulst's logistic growth have been studied by many researchers (cf. (Fan & Kuang, 2004), (Xiao & Ruan, 2001) (Hwang, 2003), (Bohner, Fan & Zhang, 2006)), there is no article on the influence of fear effect on the dynamics of ratio-dependent predator-prey model. The system (2.1) bears the potential for its merit on several counts. First, the rightful analytical proofs concerning boundedness, dissipativeness and persistence of the model system as well provided in the text affirm good foundation. Secondly, both the local and the global stability analysis for the ecologically feasible equilibrium position are carried out theoretically and duly validated numerically in order to uphold the merit of the investigation. Thirdly, the model switches off from stable phase to experience limit cycle and Hopf-bifurcation with respect to key model parameter m . This feature adds another novelty of the work undertaken. Fourth merit counts the influence of fear factor on the growth rate of the entire population under investigation and bears the destabilizing potential. Finally, the intra-specific

competition deserves its need for rectification of the system dynamics as it may control instability of the model system. One may add further that appropriate level of intra-specific competition accomplishes the predation pressure on prey resulting in the reduction of mortality risk of predator species.

8. References

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