

Stability Analysis of prey-Predator Model with Constant Harvesting of Prey Species.

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ABSTRACT:

We studied the stability analysis of prey-predator model with constant harvesting of prey-species. The basic mathematical model is described by the couple of differential equation. The Holling type function -II response is included in the prey-predator interaction. The well posed ness of the model is explored. The bounded ness property and persistence is also addressed. The local stability analysis is studied at all equilibrium points. The global dynamics is also studied and shown that the system is asymptotically stable. Further numerical simulation is executed in support of stability analysis.

Keywords: prey, predator, stability analysis, numerical examples

Mathematics Subject Classification:34DXX

Journal of Economic Literature (JEL):C62, C63

1. INTRODUCTION

Mathematical models of population growth are highly significant in nature. Nature invites many challenges to human race. The ecological predictions are quite natural to describe the stability of ecosystem. Prey-predator models are building blocks of Eco system. It Is the significant relationship in ecology Many researchers' attention has been capture in this interaction. The Dynamics of prey-predator model is explored by Carlos [2], Freedman [4], Kot [6], Lakshmi Narayan [7], Lokta [8], May.R.M [11], Murry [12,13], lima [10], Ranjith Kumar

[15] and Sita Rambabu [18]. The models with different holling time functional responses are also included in the study. Later harvesting of prey-predator models are also explored by [1,3,16].

The dynamics of this relationship can help us to protect the diversity of species in large scale. An important component in this relation is functional response of the predator. The

The detailed study of Holling type response in ecological models are dealt by [14, 17]and Authors [1,5,9] explore the prey-predator models with Holling type functional responses. Later harvesting effort is also included in prey- predator dynamics with different harvesting efforts.

Liu, W [9] studied the bifurcation analysis Michaelis- mention type harvesting models. Xiao [19] analysed the constant harvesting models. In spite of the above models, we proposed the constant harvesting in prey species with Holling type functional response.

$1 + kN_1$ and studied the dynamics of the model includes the boundedness properties, long term behaviour of the system, stability analysis at co-existing state. Finally, the analytical results are supported by numerical simulation.

2. Mathematical Model

The mathematical model equations when constant harvesting function is taken for investigation when harvesting on the prey species is given by following couple of differential equations.

$$\begin{aligned}\frac{dN_1}{dt} &= a_1N_1 - \alpha_{11}N_1^2 - \frac{\alpha_{12}N_1N_2}{1+kN_1} - H \\ \frac{dN_2}{dt} &= a_2N_2 - \alpha_{22}N_2^2 + \frac{\alpha_{21}N_1N_2}{1+kN_2}\end{aligned}\quad (2.1)$$

$$\text{With initial conditions } N_1(0) = N_{10} > 0 \text{ \& } N_2(0) = N_{20} > 0 \quad (2.2)$$

The following notations are adopted.

N_1 & N_2 : Prey and predator population, a_1, a_2 : Growth rates of prey and predator population

α_{11} : Internal competition of prey species, α_{22} : Internal competition among predator species. α_{12} & α_{21} : prey-predator interaction rates. H is constant harvesting.

2.1. Bounded ness and positivity of the model

We proved that solutions of system of equations (2.1) along with the initial conditions (2.2) are bounded and possess positive solution in R_+^2 .

To prove the results, we use the following two lemmas.

Lemma1: if $a, b > 0$ and $\frac{dx}{dt} \leq (\geq)x(t)(a - bx(t))$ with $x(0) > 0$ then

$$\log_{t \rightarrow \infty} \sup x(t) \leq \frac{a}{b} \quad (\log_{t \rightarrow \infty} \inf x(t) \geq \frac{a}{b})$$

Lemma2: if $a, b > 0$ and $\frac{dx}{dt} \leq x(t)(a - bx(t))$ with $x(0) > 0$, then for all $t \geq 0$

$$x(t) \leq \frac{a}{b - ce^{-at}} \text{ with } c = b - \frac{a}{x(0)} \text{ in particular } x(t) \leq \max\{x(0), \frac{a}{b}\} \text{ for all } t \geq 0$$

Theorem 2.1.1: All the solutions $N_1(t)$ & $N_2(t)$ of the system (2.1) are positive i.e., $N_1(t) > 0$ & $N_2(t) > 0$

Proof: From the system of equations (2.1) the prey equation is given by

$$\frac{dN_1}{dt} = N_{10} \cdot \int_0^t \left(a_1N_1 - \alpha_{11}N_1^2 - \frac{\alpha_{12}N_1N_2}{1+kN_1} - H \right) dt > 0$$

it follows that, $N_1(t) > 0$.

from the predator equations we have $\frac{dN_2}{dt} = a_2N_2 - \alpha_{22}N_2^2 + \frac{\alpha_{21}N_1N_2}{1+kN_2}$

$$\frac{1}{N_2} \frac{dN_2}{dt} = a_2 - \alpha_{22}N_2 + \frac{\alpha_{21}N_1}{1+kN_2} \text{ which implies}$$

$$N_2(t) = N_{20} e^{\int_0^t (a_2 - \alpha_{22}N_2 + \frac{\alpha_{21}N_1}{1+kN_2}) dt} > 0$$

hence $N_2(t) > 0$ for all $t \geq 0$. Thus, all the solutions $N_1(t)$ & $N_2(t)$ are positive.

Theorem 2.1.2: All the solutions $N_1(t)$ & $N_2(t)$ of the system (2.1) are bounded for all $t \geq 0$.

Proof: From (2.1) the prey equation is given by

$$\frac{dN_1}{dt} = a_1 N_1 - \alpha_{11} N_1^2 - \frac{\alpha_{12} N_1 N_2}{1 + k N_1} - H \leq N_1 (a_1 - \alpha_{11} N_1)$$

using lemma2 where both a_1 & $\alpha_{11} > 0$

$$\text{And } N_1(t) \leq \frac{a_1}{\alpha_{11} - c e^{-a_1 t}} \quad \text{and} \quad c = \alpha_{11} - \frac{a_1}{N_{10}}$$

$$\text{In particular } N_1(t) \leq \max \left\{ N_{10}, \frac{a_1}{\alpha_{11}} \right\} = M_1 \text{ for all } t \geq 0. \quad (2.1.2.1)$$

From the predator equation from (2.1) we have equations

$$\frac{dN_2}{dt} = a_2 N_2 - \alpha_{22} N_2^2 + \frac{\alpha_{21} N_1 N_2}{1 + k N_2} \leq N_2 \left(a_2 + \frac{\alpha_{21} N_1}{1 + k N_2} - \alpha_{22} N_2 \right) \text{ again, using lemma 2}$$

$$\text{Which implies } N_2 \left(a_2 + \frac{\alpha_{21} N_1}{1 + k N_2} - \alpha_{22} N_2 \right) \leq N_2 \left(a_2 + \frac{\alpha_{21} M_1}{1 + k N_2} - \alpha_{22} N_2 \right) \text{ from equation (2.1.2.1)}$$

$$\text{In particular } N_2(t) \leq \max \left\{ N_{20}, \frac{a_2(1 + k N_{20}) + \alpha_{21} M_1}{\alpha_{22}(1 + k N_{20})} \right\} = M_2 \text{ for all } t \geq 0. \quad (2.1.2.2)$$

Hence the system (2. 1) possesses bounded solutions.

2.3. PERMEANCE:

The long-term behaviour of the dynamical system in particular the positive solutions of the system approach the boundary of the positive orthant. If the system includes two species the positive solutions approach the boundary of positive quadrant (two-dimensional space) and for three species the positive solutions approach the boundary of positive octant (three-dimensional space).

The permeance of the system exist if there exist two positive constants η_1 & η_2 such that each positive solutions of $N_1(t)$ & $N_2(t)$ with initial conditions N_{10} & N_{20} in R_+^2 satisfies,

$$\min \{ \lim_{t \rightarrow \infty} \inf N_1(t, N_{10}, N_{20}), \lim_{t \rightarrow \infty} \inf N_2(t, N_{10}, N_{20}) \} \geq \eta_1 \quad \text{and}$$

$$\max \{ \lim_{t \rightarrow \infty} \sup N_1(t, N_{10}, N_{20}), \lim_{t \rightarrow \infty} \sup N_2(t, N_{10}, N_{20}) \} \leq \eta_2$$

Theorem 2.3.1. The system (2.1) is permanent if $a_2 \alpha_{22} (1 + k M_1) > \alpha_{12} [a_2 (1 + k M_1) + \alpha_{21} M_1]$

Proof: From the system of equations (2.1) the prey equation is given by

$$\frac{dN_1}{dt} = a_1 N_1 - \alpha_{11} N_1^2 - \frac{\alpha_{12} N_1 N_2}{1 + k N_1} - H \geq N_1 (a_1 - \alpha_{11} N_1 - \alpha_{12} N_2) \geq N_1 \left(a_1 - \alpha_{11} N_1 - \frac{\alpha_{12} \{a_2(1 + k M_1) + \alpha_{21} M_1\}}{\alpha_{22}(1 + k M_1)} \right)$$

using the bounded ness property from equation (2.1.2.2)

$$\text{Hence } \frac{dN_1}{dt} \geq N_1 (L_1 - \alpha_{11} N_1) \quad \text{where } L_1 = a_1 - \frac{\alpha_{12} \{a_2(1 + k M_1) + \alpha_{21} M_1\}}{\alpha_{22}(1 + k M_1)}$$

$$\text{Using lemma1 } \lim_{t \rightarrow \infty} \inf N_1(t) \geq \frac{L_1}{\alpha_{11}} \quad \text{and} \quad \lim_{t \rightarrow \infty} \sup N_1(t) \leq \frac{a_1}{\alpha_{11}} \text{ from equation (2.1.2.1)}$$

From the system of equations (2.1) the prey equation is given by

$$\frac{dN_2}{dt} = a_2 N_2 - \alpha_{22} N_2^2 + \frac{\alpha_{21} N_1 N_2}{1 + k N_1} \geq N_2 (a_2 + \alpha_{21} N_1 - \alpha_{22} N_2) \geq N_2 (a_2 + \alpha_{21} M_1 - \alpha_{22} N_2)$$

$$\text{Using lemma1 } \lim_{t \rightarrow \infty} \inf N_2(t) \geq \frac{L_2}{\alpha_{22}} \quad \text{where } L_2 = a_2 + \alpha_{21} M_1 \quad \text{and}$$

$$\lim_{t \rightarrow \infty} \sup N_2(t) \leq \frac{a_2(1 + k N_{10}) + \alpha_{21} M_1}{\alpha_{22}(1 + k N_{10})} \text{ from equation (2.1.2.2)}$$

Now choose $\eta_1 = \min \left(\frac{L_1}{\alpha_{11}}, \frac{L_2}{\alpha_{22}} \right)$ and $\eta_2 = \max \left(\frac{a_1}{\alpha_{11}}, \frac{a_2(1 + k N_{10}) + \alpha_{21} M_1}{\alpha_{22}(1 + k N_{10})} \right)$ we get the permanence of the system (2.1).

2.4. EXISTANCE OF EQUILIBRIA:

The lowest energy state of the system which is also stable is occurs at equilibrium points. Solve the system of equations (2.1) we get the following possible equilibrium points.

I. The Extinct state $E_1: \bar{N}_1 = 0, \bar{N}_2 = 0$ (2.4.1)

II. Semi Extinct: One of the species will survive and other one is washed out.

Case (A) : Prey species exist and predator washed out.

To solve for the value of $\bar{N}_1$ from equation (2.1) we get the quadratic equation given by

$$\alpha_{11}N_1^2 - a_1N_1 + H = 0 \quad (2.4.2)$$

Solving the above equation (2.4.2) we get two roots for $\bar{N}_1$ which is given by

$$\bar{N}_{1L} = \left(\frac{a_1}{2\alpha_{11}} \right) - \sqrt{(a_1)^2 - 4\alpha_{11}H} \quad (2.4.3)$$

$$\bar{N}_{1H} = \left(\frac{a_1}{2\alpha_{11}} \right) + \sqrt{(a_1)^2 - 4\alpha_{11}H} \quad (2.4.4)$$

The equilibrium point exists only if $(a_1)^2 > 4\alpha_{11}H$

Hence the equilibrium points are $E_2: (\bar{N}_{1L}, 0) \& (\bar{N}_{1H}, 0)$ (2.4.5)

Case(B) : Prey species washed out & predator species exist .

$$E_3: \bar{N}_1 = 0, \bar{N}_2 = \frac{a_2}{\alpha_{22}} \quad (2.4.6)$$

III: Two species are survived

Solve the system of equations (2.1) we get the by quadric equation in N_2 is given by

$$a\lambda^4 + b\lambda^3 + c\lambda^2 + d\lambda + e = 0 \quad (2.4.7)$$

Here

$$a = \alpha_{11}\alpha_{22}^2k^3, b = 2k^2\alpha_{11}\alpha_{22}(\alpha_{22} - ka_2)$$

$$c = \alpha_{21}\alpha_{11}\alpha_{22}k + k\alpha_{11}(\alpha_{22}^2 + a_2^2k^2 - 4ka_2\alpha_{22}) - (a_1 - H)k^2\alpha_{12}\alpha_{22}$$

$$d = -k\alpha_{21}(a_1 - H)(\alpha_{22} - ka_2) + \alpha_{21}\alpha_{11}\alpha_{22} - ka_2\alpha_{21}\alpha_{11} + 2k\alpha_{11}a_2^2 - 2ka_2\alpha_{11}\alpha_{22} + \alpha_{12}$$

$$e = -(a_1 - h)\alpha_{21}^2 - ka_2\alpha_{21}(a_1 - H) + a_2\alpha_{11}\alpha_{21} - k\alpha_{11}a_2^2 \quad (2.4.8)$$

On solving equation (2.4.7), four possible roots exist for $\bar{N}_2$. The second equilibrium point $\bar{N}_1$ is obtained from the following equation.

$$\bar{N}_1 = \frac{(1+k\bar{N}_1)(\alpha_{22}\bar{N}_2 - a_2)}{\alpha_{21}} \quad (2.4.9)$$

The Four possible equilibria for this case are obtained by solving equation (2.4.7) and for each value of $\bar{N}_2$ and corresponding value $\bar{N}_1$ is obtained from equation (2.4.9).

2.5. STABILITY ANALYSIS AT EQUILIBRIA.

Local dynamics of the system calculated as follows.

Find the Jacobean matrix for the system (2.1) give as follows.

$$J = \begin{bmatrix} J_{11} & J_{12} \\ J_{21} & J_{22} \end{bmatrix}$$

Here

$$\begin{aligned} J_{11} &= \frac{\partial f_1}{\partial N_1}, J_{12} = \frac{\partial f_1}{\partial N_2}, \\ J_{21} &= \frac{\partial f_2}{\partial N_1}, J_{22} = \frac{\partial f_2}{\partial N_2} \end{aligned} \quad (2.5.1)$$

Here

$$\begin{aligned} f_1(N_1, N_2) &= a_1 N_1 - \alpha_{11} N_1^2 - \frac{\alpha_{12} N_1 N_2}{1 + k N_1} - H \\ f_2(N_1, N_2) &= a_2 N_2 - \alpha_{22} N_2^2 + \frac{\alpha_{21} N_1 N_2}{1 + k N_2} \end{aligned} \quad (2.5.2)$$

Calculate the Jacobean matrix i.e.

$$J = \begin{bmatrix} -\alpha_{11} N_1 + \frac{k \alpha_{12} N_2}{(1 + k N_1)^2} & -\frac{\alpha_{12} N_1}{1 + k N_1} \\ \frac{\alpha_{21} N_2}{(1 + k N_1)^2} & -\alpha_{22} N_2 \end{bmatrix} \quad (2.5.3)$$

Calculate the eigen equation for the above matrix given by $\det(J - \lambda I) = 0$.

Find the eigen values at each equilibrium point to find the nature of the system.

Case (i) $E_1(0,0)$ is unstable.

Case (ii): case A: The eigen equation for E_2 is $\lambda^2 + (\alpha_{11} + k \alpha_{21}) \bar{N}_1 \lambda = 0$, where roots are $\lambda = 0$ & $\lambda = -(\alpha_{11} + k \alpha_{21}) \bar{N}_1$ hence the system neutrally stable.

Case (ii): case B: The Eigen equation for E_3 is $\lambda^2 + (\alpha_{22} - k \alpha_{12}) \bar{N}_2 \lambda - k \alpha_{12} \alpha_{22} \bar{N}_2^2 = 0$,

The system is unstable since the product of the root is negative i.e., $-k \alpha_{12} \alpha_{22} \bar{N}_2^2$

Case (iii): E_4 : Two species survived.

The eigen equation is given by $a \lambda^2 + b \lambda + c = 0$ (2.5.4)

where $a=1$,

$$\begin{aligned} b &= \left(\frac{-k \alpha_{12} N_2}{(1 + k N_1)^2} + \frac{k \alpha_{21} N_1}{(1 + k N_2)^2} + \alpha_{11} N_1 + \alpha_{22} N_2 \right), \\ c &= \left(\frac{k^2 \alpha_{12} \alpha_{21} N_1 N_2}{(1 + k N_1)^2 (1 + k N_2)^2} - \frac{k \alpha_{12} \alpha_{22} N_2^2}{(1 + k N_1)^2} + \alpha_{11} \alpha_{22} N_1 N_2 + \frac{k \alpha_{21} \alpha_{11} N_1^2}{(1 + k N_2)^2} + \frac{\alpha_{12} \alpha_{21} N_1 N_2}{(1 + k N_1)(1 + k N_2)} \right) \end{aligned} \quad (2.5.5)$$

The system is stable if the sums of roots are negative, and products of roots are positive i.e

$$\begin{aligned} \text{If } \alpha_{11} N_1 + \alpha_{22} N_2 + \frac{k \alpha_{21} N_1}{(1 + k N_2)^2} &> \frac{k \alpha_{12} N_2}{(1 + k N_1)^2} \text{ and} \\ \alpha_{11} \alpha_{22} N_1 N_2 + \frac{k \alpha_{21} \alpha_{11} N_1^2}{(1 + k N_2)^2} + \frac{\alpha_{12} \alpha_{22} N_1 N_2}{(1 + k N_1)(1 + k N_2)} &+ \frac{k^2 \alpha_{12} \alpha_{21} N_1 N_2}{(1 + k N_1)^2 (1 + k N_2)^2} > \frac{k \alpha_{12} \alpha_{22} N_2^2}{(1 + k N_1)^2} \end{aligned} \quad (2.5.6)$$

E_4 is locally asymptotically stable condition (2.5.6) is satisfied otherwise unstable.

2.6. GLOBAL DYNAMICS:

Theorem 2.6.1: The system (2.1) is globally asymptotically stable at co-existing state E_4 .

Proof: Choose the following Lyapunov's function

$$V(\bar{N}_1, \bar{N}_2) = l_1(N_1 - \bar{N}_1) - \bar{N}_1 \log\left(\frac{N_1}{\bar{N}_1}\right) + l_2\left[(N_2 - \bar{N}_2) - \bar{N}_2 \log\left(\frac{N_2}{\bar{N}_2}\right)\right] \quad (2.6.1)$$

Calculate time derivative for the above equation (2.6.1), we get.

$$\frac{dV}{dt} = l_1 \frac{dN_1}{dt} \left[1 - \frac{\bar{N}_1}{N_1}\right] + l_2 \frac{dN_2}{dt} \left[1 - \frac{\bar{N}_2}{N_2}\right] \quad (2.6.2)$$

$$= l_1[N_1 - \bar{N}_1] \left[(a_1 - \alpha_{11}N_1 - \frac{\alpha_{12}N_2}{1+kN_1} - H) \right] + l_2[N_2 - \bar{N}_2] \left[a_2 - \alpha_{22}N_2 + \frac{\alpha_{21}N_1}{1+kN_2} \right] \quad (2.6.3)$$

By proper choice of

$$a_1 = \alpha_{11}\bar{N}_1 + \frac{\alpha_{12}\bar{N}_2}{1+k\bar{N}_1} + H, a_2 = \alpha_{22}\bar{N}_2 - \frac{\alpha_{21}\bar{N}_1}{1+k\bar{N}_2} \& l_1 = \frac{1+k\bar{N}_1}{\alpha_{12}}, l_2 = \frac{1+k\bar{N}_1}{\alpha_{21}}$$

$$\text{We get } \frac{dV}{dt} = -\alpha_{11} \frac{1+k\bar{N}_1}{\alpha_{12}} (N_1 - \bar{N}_1)^2 - \frac{\alpha_{22}(1+k\bar{N}_1)}{\alpha_{21}} (N_2 - \bar{N}_2)^2 \quad (2.6.4)$$

Clearly $\frac{dV}{dt} < 0$ therefore the system is globally asymptotically stable at co-existing state.

3. NUMERICAL SIMULATION:

Example 3.1: Let $a_1=1, a_2=1, k=0.5, \alpha_{11}=0.1, \alpha_{12}=0.1, \alpha_{21}=0.05, \alpha_{22}=0.5, N_1=25, N_2=10$.

Fig A: represents time series plot B: Represents phase portrait.

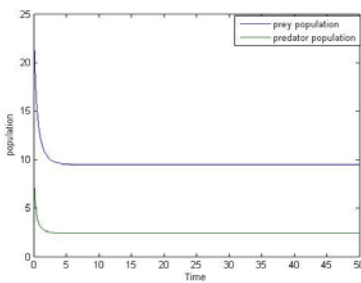


Fig 3.1(A)

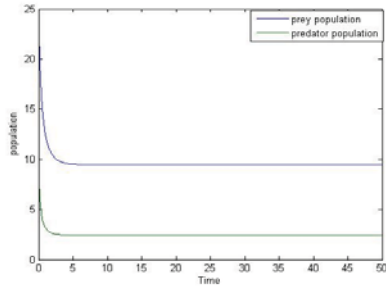


Fig 3.1(B)

3.1(A) : with (i) $H = 0.1$, (ii) $H = 0.01$ (iii) $H = 1$ (iv) $H = 0.05$ (v) $H = 2$

Stable solutions converging to fixed equilibrium point $E(9.2)$

The range of $0.5 < k < 500$ the prey & predator population range varies from

$$8 \leq N_1 \leq 10 \& 2 \leq N_2 \leq 2.2$$

Fig 3.1(B): with (i) $H \geq 2.3$

The system becomes stable to the fixed-point E (0,2)

Example 3.2: Let $a_1=1.5$, $a_2=2$, $k=5$, $\alpha_{11}=0.1$, $\alpha_{12}=0.2$, $\alpha_{21}=0.03$, $\alpha_{22}=0.1$, $N_1=25$, $N_2=10$.

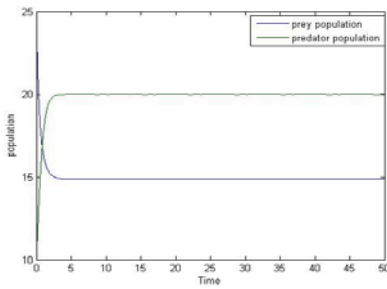


Fig 3.2(A)

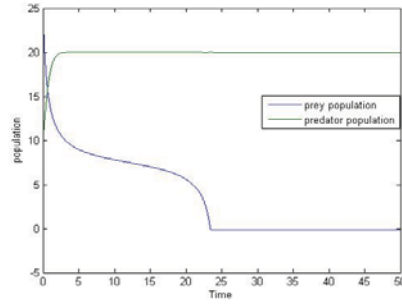


Fig 3.2(B)

Fig 3.2(A): with $0.01 \leq H \leq 4.5$

Stable solutions converging to fixed equilibrium point E (14,20)

The range of $5 < k < 500$ the prey & predator population range varies from

$$10 \leq N_1 \leq 15 \text{ \& } 19 \leq N_2 \leq 20$$

Fig 3.2(B): with (i) $H \geq 5$ Stable solutions converging to fixed equilibrium point E (0,20)

4. CONCLUSION

We consider a mathematical model of prey- predators' interactions with constant harvesting on prey species for investigation. The mathematical model with prey predator dynamics was studied and prove that the eco-system is stable for co-existing state. The properties of the model were studied like positivity, boundedness and permeance of the system. The stability analysis of the model was discussed at possible equilibrium points.

Equilibrium points	Nature of the system with linear type harvesting
$E_1: \bar{N}_1 = 0, \bar{N}_2 = 0$	Un stable
$E_2: \bar{N}_1 \neq 0, \bar{N}_2 = 0$	Neutrally stable
$E_3: \bar{N}_1 = 0, \bar{N}_2 \neq 0$	Un stable
$E_4: \bar{N}_1 \neq 0, \bar{N}_2 \neq 0$	Asymptotically stable if $\alpha_{11}N_1 + \alpha_{22}N_2 + \frac{k\alpha_{21}N_1}{(1+kN_2)^2} > \frac{k\alpha_{12}N_2}{(1+kN_1)^2} \text{ and}$ $\alpha_{11}\alpha_{22}N_1N_2 + \frac{k\alpha_{21}\alpha_{11}N_1^2}{(1+kN_2)^2} + \frac{\alpha_{12}\alpha_{21}N_1N_2}{(1+kN_1)(1+kN_2)} +$ $\frac{k^2\alpha_{12}\alpha_{21}N_1N_2}{(1+kN_1)^2(1+kN_2)^2} > \frac{k\alpha_{12}\alpha_{22}N_2^2}{(1+kN_1)^2}$

Table 4.1

The global stability analysis is also studied and prove that the system is asymptotically stable. Further Numerical simulation is performed in support of analytical results. The harvesting effort results are placed below.

Examples	Results with constant harvesting
Example 3.1	Stable solutions converging to fixed equilibrium point E (9,2) When (i) $H = 0.1$, (ii) $H=0.01$ (iii) $H=1$ (iv) $H=0.05$ (v) $H=2$ prey population is extinct. when (i) $H \geq 2.3$ Stable solutions converging to fixed equilibrium point E (0,2).
Example 3.2	with $0.01 \leq H \leq 4.5$ Stable solutions converging to fixed equilibrium point E (14,20). prey population is extinct. with (i) $H \geq 5$ Stable solutions converging to fixed equilibrium point E (0,20).

Table 4.2

The constant effort range for extinction of prey population is identified using numerical simulation.

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