

Application of GMDH-type ANN model for Prediction of Total Fertility Rate: A case study of India

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Abstract

The present study aimed to compare the group method of data handling type artificial neural network (GMDH-type ANN) model and autoregressive integrated moving average (ARIMA) model for prediction of India's total fertility rate (TFR). In this study, time series TFR data was collected from the period 1995–96 to 2019–2020. The ARIMA (1,2,2) model was found to have lower Akaike's information criterion (AIC) and Bayesian information criterion (BIC) values, making them more acceptable. After that, GMDH-type ANN model was fitted and to check the adequacy of these two models using different error measures such as the root mean square error (RMSE), mean absolute error (MAE), mean absolute percentage error (MAPE) and scatter index (SI). The GMDH-type ANN model is superior over ARIMA (1,2,2) model. Thus, the GMDH-type ANN model was used to predict the TFR of India for the next 10 years and the results indicated that the TFR will decline in upcoming years. The government will use the information regarding the prediction of TFR to allocate the forthcoming resources and also make decisions for people's welfare.

Keywords: Total Fertility Rate, ARIMA, GMDH-type ANN,

Mathematics Subject Classification: 62J12, 62G99

Journal of Economic Literature (JEL) Classification: Q12, D24

1. INTRODUCTION

India's fertility rate has significantly declined, reaching replacement levels in 18 of its 29 states (Jejeebhoy *et al.*, 2020). This reduction has been driven by a variety of factors, with marriage, contraception, and abortion playing key roles (Singh *et al.*, 2022). The decline in fertility has had both positive and negative effects, including a reduction in maternal mortality and a shift in the population pyramid (Radkar, 2020). However, there is still a need for improvements in sexual and reproductive health services, particularly in terms of access and quality. Despite these challenges, there is potential

for significant economic returns from family planning investments, particularly if the fertility rate can be further reduced (Goli *et al.*, 2023).

The TFR is a measurement of the typical lifetime number of children that a woman would have if she experienced the current age-specific fertility rates during her reproductive period (Singh *et al.*, 2020; Tiwari *et al.*, 2020). It is a measure of the potential for population growth (or decline). In India, the TFR has been declining for several decades. In 1951, the TFR was 6.0 children per woman. By 2015-16, it had declined to 2.2 children per woman. The decline in TFR has had a significant impact on India's population growth rate. In the 1950s, India's population was growing at a rate of over 2% per year. By the 2010s, the population growth rate had declined to less than 1% per year (Singh and Singh, 2023). This decline in population growth is expected to continue in the coming years. The decline in TFR has also had a significant impact on India's age structure which has several implications- First, it will lead to a slower rate of population growth. This will give India more time to address its development challenges, such as poverty and unemployment. Second, the decline in TFR will lead to an aging population. This will put a strain on India's social welfare system, which is already struggling to meet the needs of the elderly. Third, the decline in TFR will lead to a change in India's labour force. The declining number of young people will make it more difficult for businesses to find workers. This could lead to a shortage of labour and a slowdown in economic growth (Purkayastha *et al.*, 2024).

The TFR is 2.1 children per woman, which closely approximates the equivalent replacement level fertility rate of 2.1 children per woman. This means that each woman in India has approximately two children, which is just enough to replace the population. According to the National Family Health Survey 2021, the national TFR is 2.0, which is below the replacement level of 2.1. This decline is attributed to several factors such as rising female education and labour force participation, increased access to family planning services and changes in social norms and values (Goli *et al.*, 2023).

Several researchers considered ARIMA models for the future prediction of fertility rates (Waseem & Yasmeen, 2016; Bolanle & Oluwadare, 2017; Raziq *et al.*, 2021; Karunanidhi & Sasikala, S. 2023). ARIMA models are not appropriate for long-term future prediction of non-linear data but GMDH-type ANN model has this applicability (Adeyinka and Muhajarine, 2020). However, to increase the accuracy of predictive models, some authors have proposed artificial intelligence techniques like deep learning (Al-Maqaleh *et al.*, 2016; Kihoro *et al.*, 2004; Aladag, 2011) and machine learning (Hsieh *et al.*, 2018; Sakr *et al.*, 2017; Son *et al.*, 2010). The GMDH-type ANN technique has been shown to enhance forecasting in the domains of manufacturing, agriculture, economics and urban planning. The effectiveness of multilayer perceptron network, a widely used ANN and GMDH-type ANN techniques were assessed for predicting the comprehensiveness and workability of concrete (Ghazanfari, *et al.*, 2017) and their research revealed that GMDH-type ANN produced more accurate outcomes. A comparative study was conducted by researchers to evaluate the predictive performance of GMDH-type ANN technique in comparison to ARIMA and Holt's techniques (Mishra *et al.*, 2023). The researchers determined that the GMDH-type ANN technique has greater prediction capabilities compared to other models.

The present study aims to determine the stationarity of a given data set of TFR using the Augmented Dickey-Fuller (ADF) test to choose the most suitable ARIMA model. After that, a comparison will be made between the adequate ARIMA model and the GMDH-type ANN model.

2. MATERIALS AND METHODS

The data for the period 1995 to 2020 were collected from the source “<https://www.macrotrends.net/global-metrics/countries/IND/india/fertility-rate>”. MS-Excel, R and GMDH-type ANN shell Software were used for the data analysis.

2.1 Autoregressive Integrated Moving Average (ARIMA)

The ARIMA models, created by (Box and Jenkins, 1970) are a classic econometric technique for time series forecasting. The autoregressive moving average (ARMA) model is modified by the ARIMA. An ARMA model expressed the conditional average of Y_t as a function of both previous observations $Y_{t-1}, Y_{t-2}, \dots, Y_{t-p}$, and previous innovations, $\varepsilon_{t-1}, \varepsilon_{t-2}, \dots, \varepsilon_{t-q}$. The number of previous observations that Y_t depends on, p , is the AR degree. The number of previous innovations that Y_t depends q , is the MA degree. In general, these methodologies are denoted by ARMA (p, q). The form of the ARMA (p, q) approach is:

$$Y_t = \alpha + \beta_1 Y_{t-1} + \beta_2 Y_{t-2} + \dots + \beta_p Y_{t-p} + \varepsilon_t + \varphi_1 \varepsilon_{t-1} + \varphi_2 \varepsilon_{t-2} + \dots + \varphi_q \varepsilon_{t-q} \dots \quad (1)$$

where α is a constant term, $\beta_1, \dots, \beta_p$ autoregressive (AR) coefficients, φ_1 moving average (MA) coefficients, $Y_{t-1}, Y_{t-2}, \dots, Y_{t-p}$, AR lags corresponding to non-zero, $\varepsilon_{t-1}, \dots, \varepsilon_{t-q}$ MA lags corresponding to nonzero, MA coefficients, and degree of differencing D , if D has value 0 which means no integration.

The Box-Jenkins approach's linearity restriction has been solved by statisticians in many different kinds of methods, and in addition to nonlinear time series models, robust versions of several ARIMA models have been developed (Makridakis *et al.*, 1997). In this study, forecasting and fitting were done using a box-Jenkins' ARIMA model technique and its flowchart is shown in Figure 1.

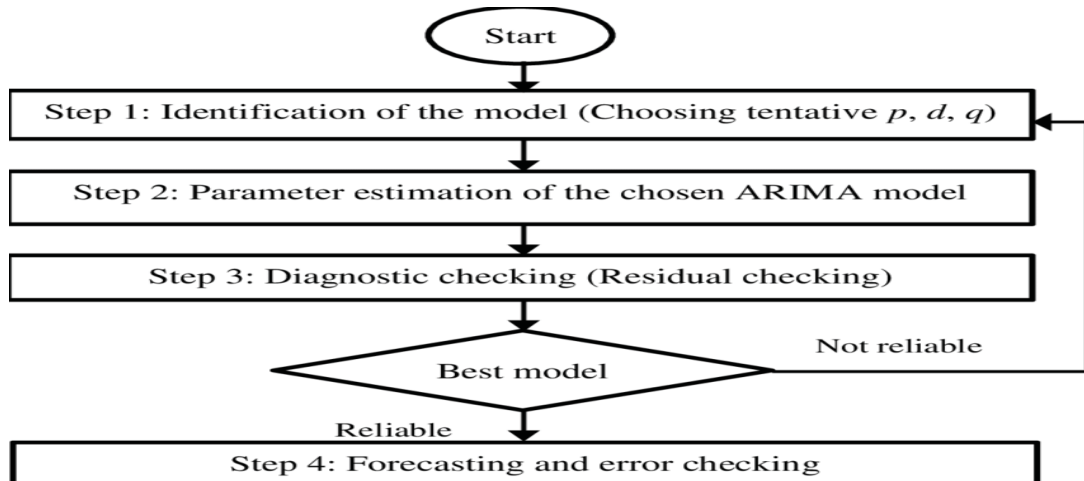


Figure. 1 Structure of the Box-Jenkins methodology

2.1.1 Test for Stationarity: Augmented Dickey-Fuller (ADF) test

To find out if a time series of data is stationary, one of the best methods is to use the ADF test. The null hypothesis (H_0) in this study states that the time series data are not stationary, while the alternative hypothesis (H_1) states that the data are stationary. The hypothesis is tested using appropriate d^{th} order data differencing and ADF test apply on the differenced model.

2.2 Group Method of Data Handling Type Artificial Neural Network (GMDH-type ANN)

The GMDH method was first used by (Ivakhnenko, 1968) to look at complex structures made up of a set of data with multiple inputs and a single output. For the GMDH network, the main goal is to create a feed-forward network function using a second-degree transfer function. The GMDH method naturally picks the best model structure based on the input factors, hidden layer neurons, and layer count.

$$\hat{y} = a_0 + \sum_{i=1}^m a_i x_i + \sum_{i=1}^m \sum_{j=1}^m a_{ij} x_i x_j + \sum_{i=1}^m \sum_{j=1}^m \sum_{k=1}^m a_{ijk} x_i x_j x_k + \dots \quad (2)$$

where x represents the input variable, a_i represents coefficients, y represents the output variable and m represents the number of observations of input variable. The process of GMDH-type ANN model is shown in Figure 2.

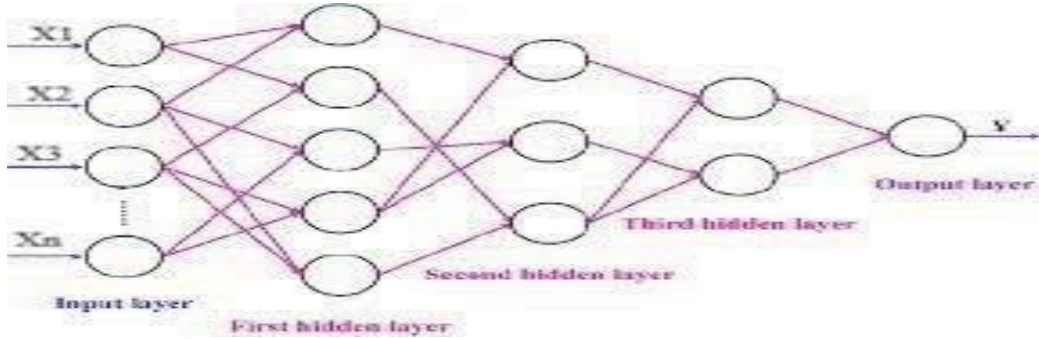


Figure. 2 GMDH- type ANN model development process

2.3 Prediction Accuracy Measures

Model selection criteria are the standards by which a model's performance for the data being studied is evaluated to ascertain if a consistent pattern in the models' performances is present (Bal *et al.*, 2016; Choi and Jeong, 2019; Koutsandreas *et al.*, 2022; Spiliotis *et al.*, 2023). The different accuracy measures (AIC and BIC) were used to determine and validate the results of various ARIMA models for predicting TFR of India. For the comparative study of GMDH-type ANN model and ARIMA model, the accuracy measures (RMSE, MAE, MAPE and SI) were used. The SI is a standardized measure of error, frequently presented as a percent. Minor values of the SI are an suggestion of superior model performance (Mentaschi *et al.*, 2013).

$$AIC = 2 \times K - 2 \times \ln L$$

$$BIC = K \times \ln N - 2 \times \ln L$$

$$RMSE = \sqrt{\frac{1}{N} \sum_{i=1}^N (y_i - \hat{y}_i)^2}$$

$$MAE = \frac{1}{N} \sum_{i=1}^N |y_i - \hat{y}_i|$$

$$MAPE = \frac{1}{N} \sum_{i=1}^N \left| \frac{y_i - \hat{y}_i}{y_i} \right|$$

$$SI = \frac{RMSE}{\bar{y}} * 100$$

where

N= number of observations, K = number for the parameters in the model, L = the likelihood function, $\hat{y}_i$ = Predicted value of y, y_i = the actual value of y, $\bar{y}$ = Mean of actual value of y

3. RESULTS AND DISCUSSION

Using the past known data of TFR for India, 10 years of future data prediction will attain with the reliable models.

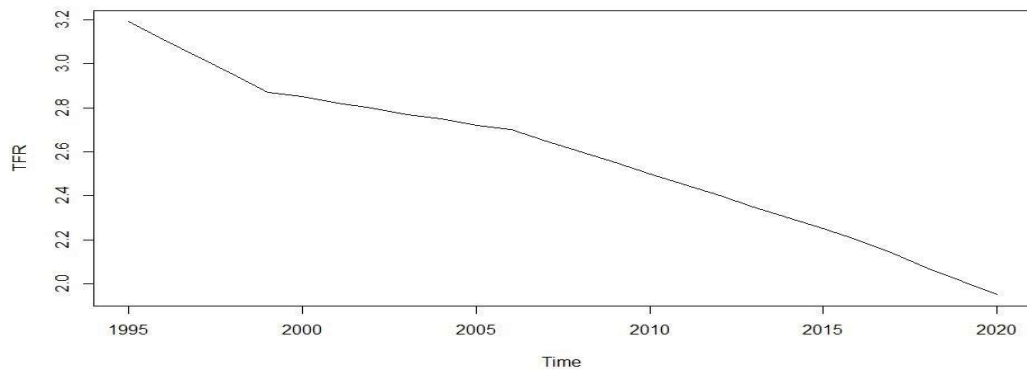


Figure. 3 Graphical presentation of TFR of India (1995-2020)

The TFR of India shows the declining trend and the given time is non-stationary (see Figure 3)

3.1 Fitting of ARIMA model

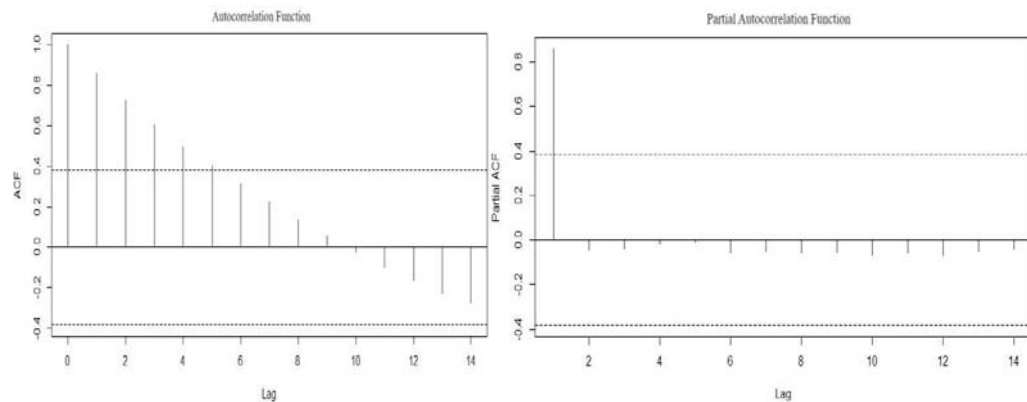


Figure. 4 ACF and PACF at the Lag= 14 for TFR of India (1995-2020)

Plots for ACF and PACF are shown in Figure 4 accordingly. The autocorrelation values show a progressive decline from the initial second-order autocorrelation coefficient to the final value. The ADF test confirmed the acceptance of the Null hypothesis, indicating that the TFR of India (1995-2020) follows non-stationarity which is shown in Table 1.

3.2 Differencing the data series of TFR to reach the stationarity

The Box-Jenkins method requires the time series data is stationary. A correlogram graphic is used to assess the stationarity of time series data. If stationarity is not attained, the differencing method is used to eliminate variations in the series, resulting in a constant mean, variance and co-variance throughout time (see Figure 5).

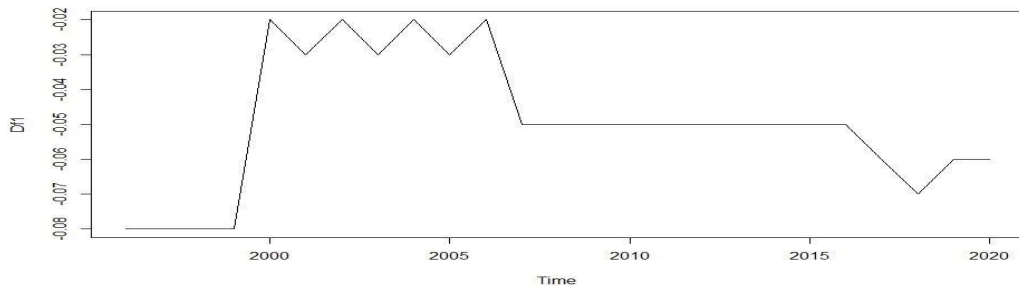


Figure. 5 First differencing of the time series plot for TFR

Figure 5 shows that the TFR data is non-stationary after first order differencing. A transferred series after second-order differencing to the TFR data to make stationary series which is shown in Figure 6 and the ACF and PACF plots after second-order differencing are shown in Figure 7.

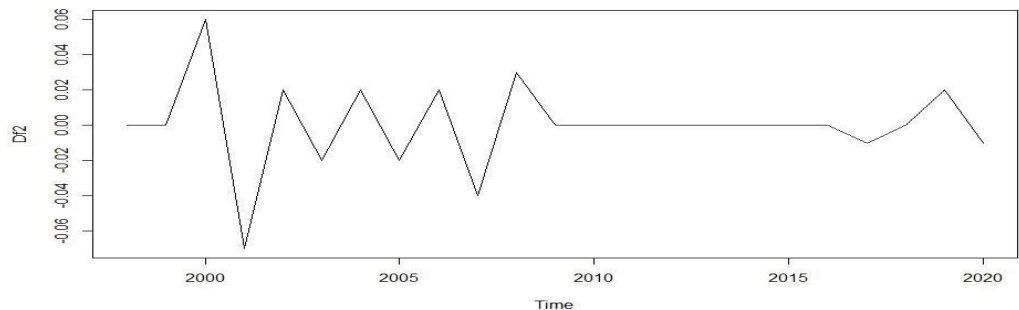


Fig. 6 Second differencing of the time series plot for TFR

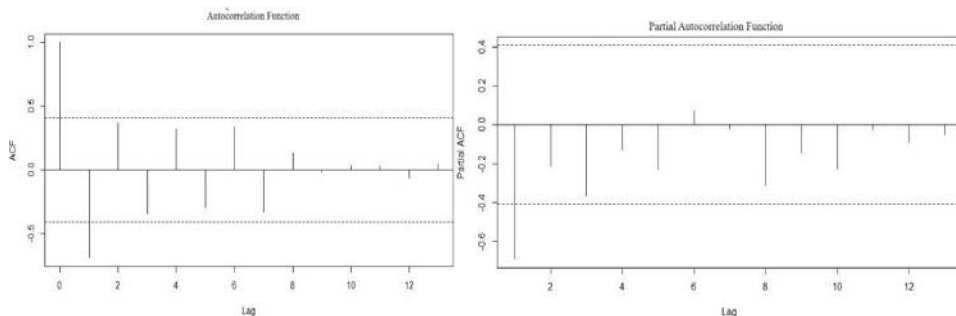


Figure. 7 ACF and PACF plots for the second differencing of the TFR data

The ADF test statistic is compared against critical values at significance levels, typically set at 5%. If the p-value is below the significance level, you can reject the null hypothesis and determine that the time series is stationary. When the p-value is above the significance level, you take the null hypothesis and arrange that the time series has a unit root (see Table1).

Table 1 ADF test to check the stationarity of TFR for India

Parameters	Time series data	1 st order differencing	2 nd order differencing
ADF -Test statistic	-2.3109	-3.3547	-7.9728
Lag order	2	2	2
p-value at 5% level of significance	0.454 (NS)	0.08407 (NS)	0.01(S)

Note: Significant (S): p-value <0.05, Non- significant (NS): p-value >0.05

From Table 1, it is concluded that the stationarity is generated by the 2nd order differentiated data in TFR at a significance level of 5%. The alternative hypothesis is accepted when the p-value is less than 0.05. Then the transformed time series data (with 2nd order differencing) is stationary. AIC and BIC values were used to determine the goodness of fit for estimated ARIMA models which were shown in the Table 2.

Table 2 ARIMA model selection criteria for TFR

Models	AIC	BIC
ARIMA (1,1,1)	-131.40	-127.75
ARIMA (1,1,2)	-130.02	-125.15
ARIMA (1,1,3)	-128.60	-122.51
ARIMA (1,1,4)	-129.48	-122.17
ARIMA (1,1,5)	-128.84	-120.31
ARIMA (1,1,6)	-128.00	-118.25
ARIMA (1,2,1)	-130.83	-127.30
ARIMA (1,2,2)	-132.85	-128.14

The ARIMA (1,2,2) model is the most suitable for the second-order differenced data of the TFR for India, as seen in Table 2. The top-performing model in this study, ARIMA (1,2,2) was selected by evaluating measures like AIC (-132.85) and BIC (-128.14) which showed the lowest values as compared to all other ARIMA models. The ARIMA (1,2,2) model follows a normal distribution with mean 0 and variance 1, the ACF and estimated models are similarly presented in Figure 8.

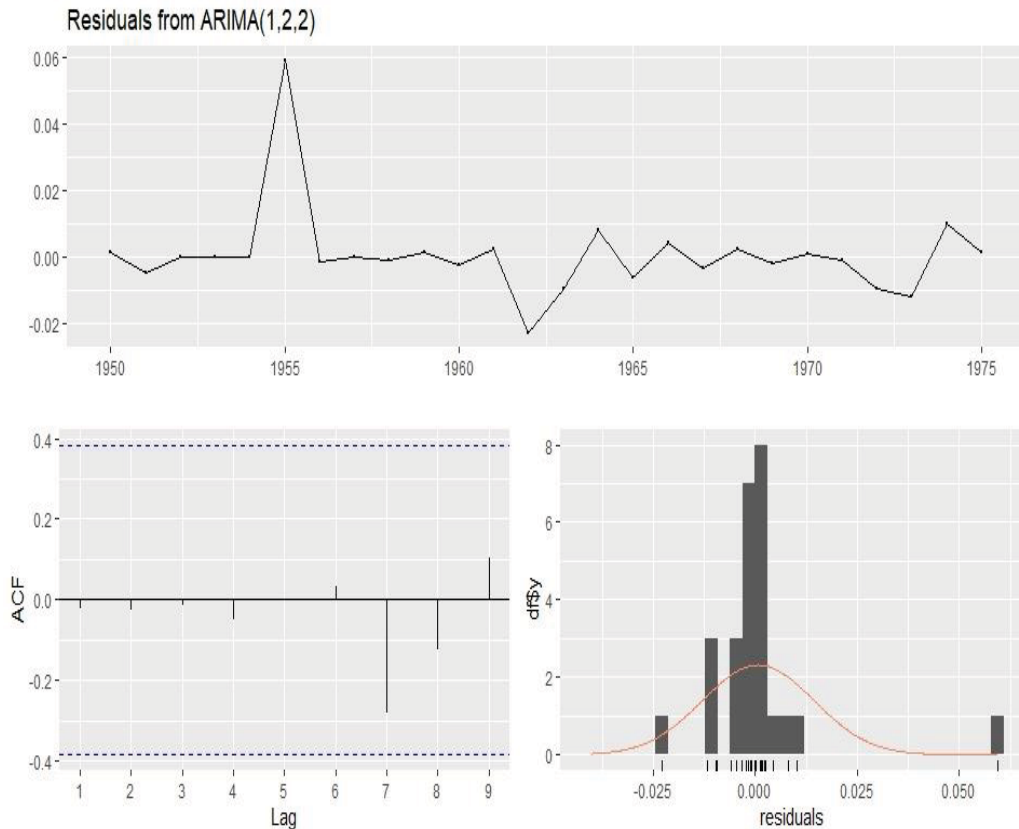


Figure. 8 Residuals plot from ARIMA (1,2,2) model with zero mean

Figure 8 shows that ARIMA (1,2,2) model for India's TFR were statistically significant with fulfilment of the normality condition of the residuals.

3.3 Fitting of GMDH-type ANN model

The TFR trend is automatically removed by the built-in time series pre-processing capabilities of the GMDH-type algorithm (GMDH Shell, 2017). With a minimum of zero lag and a maximum of 2 lags, the target variable TFR was automatically converted into a cube root. Both time and the converted TFR's lag were input variables. The network was built using the dataset randomly partitioned using the Pareto approach (Macek, 2008; Allen and Hooper, 2018)—80% was used for training (2005–2020) and 20% was the test dataset for testing model performance (1995–2004). This prevented under- or over-fitting of TFR that may arise from faulty training of the dataset. With polynomial neural networks of the GMDH type, we were able to achieve the greatest, which allowed us to create an ideal neural-type time series model, as shown in Figure 9.

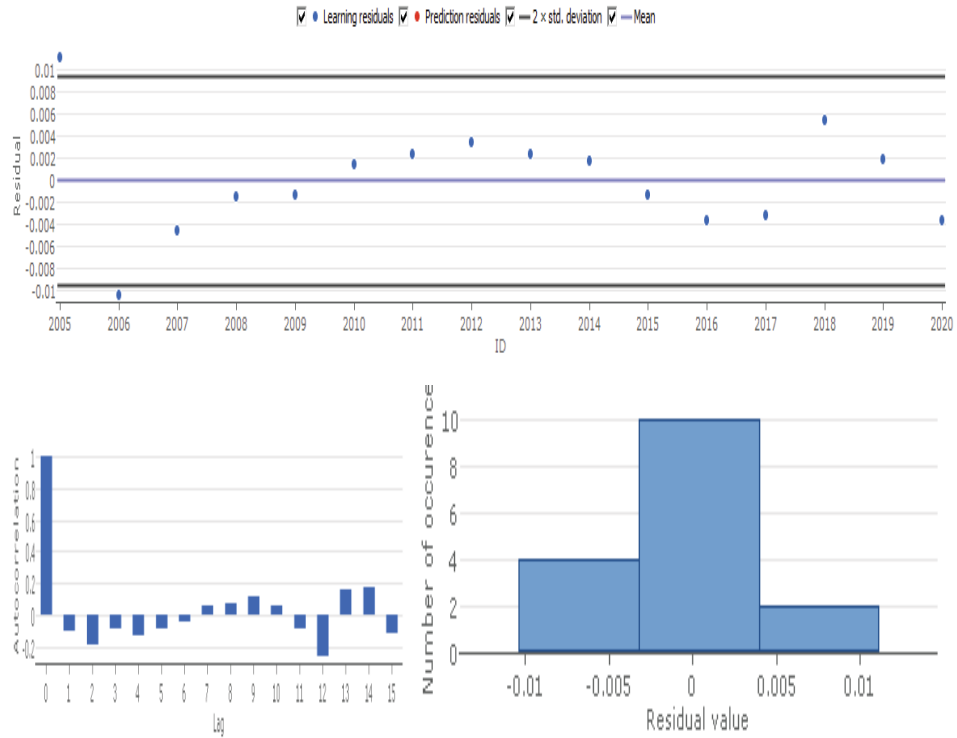


Figure. 9 Residual plots for GMDH-type ANN model for TFR of India

The adequacy of the model was additionally confirmed with a low criterion value of 2.46182 and residual plots which is shown in Figure 9, it may be concluded that the final neural architecture shows an acceptable match to the data. The equation for the GMDH-type ANN model of TFR defines the parameters and coefficients as follows:

$$Y_t = 2.46182 - N_{15} * 0.0555152 \quad (3)$$

where Y is year of forecast and N stands for neurons which is 15.

3.4 Model Comparison

The forecast issue with determining key performance indicators is assessing prediction accuracy. For prediction accuracy, however, metrics like RMSE, MAE, MAPE and SI are employed which were shown in the Table 3. The model with the lowest values of these metrics provides good fit for TFR in India.

This study compared the predictive ability of GMDH-type ANN model with traditional statistical model (ARIMA-1,2,2) in view of forecasting TFR for India from 1995 to 2030 (see Fig. 10a & b).

Table 3 Comparative performance measures of GMDH-type ANN model and ARIMA (1,2,2) model for TFR in India

Models	ARIMA	GMDH-type ANN
Best parameters	$p = 1, d = 2, q = 2$	Training set=80%, Testing set= 20%
RMSE	0.0224	0.0033
MAE	0.0155	0.0025
MAPE	0.6311	0.0956
SI	0.0087	0.0013

As shown in Table 3 the SI value (0.0013) was the lowest for the GMDH-type ANN model as compared to the ARIMA (1,2,2) model. Further comparison between the GMDH-type ANN model and the ARIMA (1,2,2) model with RMSE, MAE, and MAPE indicated that the GMDH-type ANN model's performance was better as all error measures depicted lower values (0.0033, 0.0025, and 0.0956).

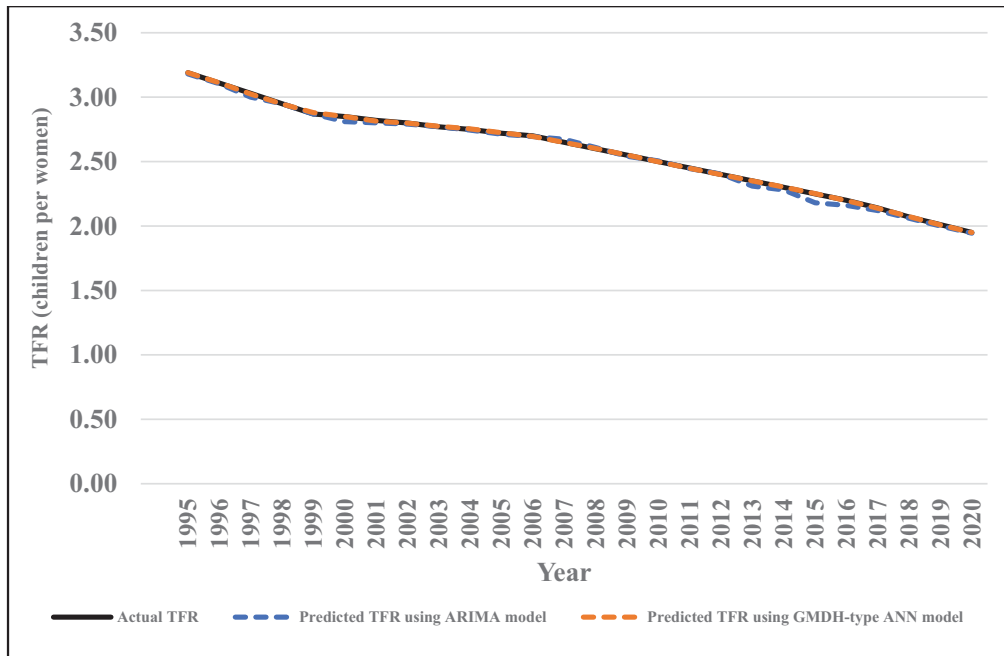


Figure. 10a Comparison of the ARIMA and GMDH-type ANN models for TFR prediction

Figure 10a shows that the predicted TFR values for the years 1995 to 2020 were exactly similar to the actual values of TFR using the GMDH-type ANN model, while using the ARIMA (1,2,2) model for the prediction, there is a little difference between the actual and predicted values of TFR. Therefore, it is shown that the GMDH-type ANN model performs better than the ARIMA (1,2,2) model in predicting TFR for India.

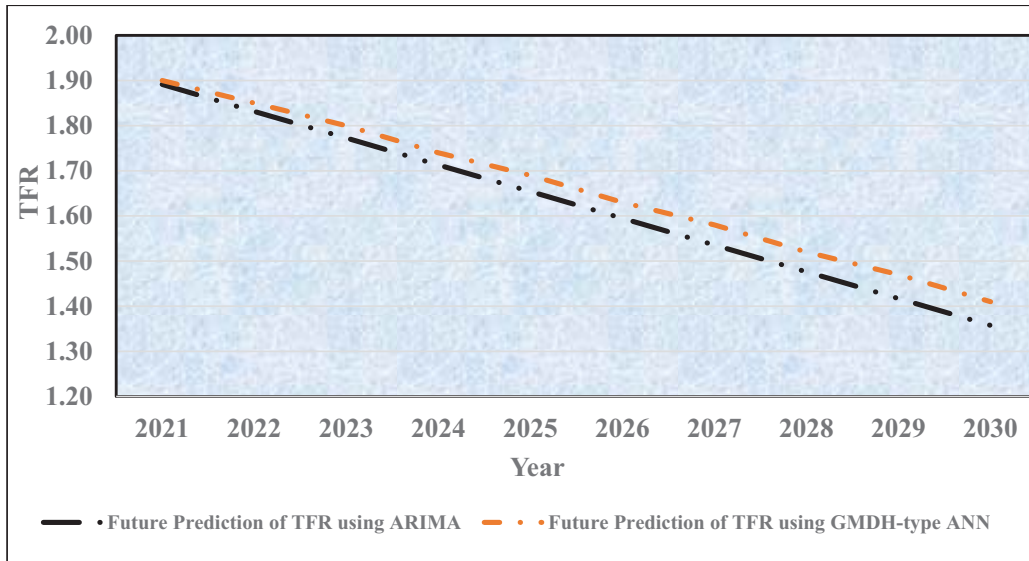


Figure. 10b Comparison of the ARIMA and GMDH-type ANN models for TFR future prediction.

From Figure 10a, the in-sample prediction of TFR from 1995 to 2020 from ARIMA (1,2,2) and GMDH-type ANN models were closed to the observed historical rates. However, the out-of sample forecasts from 2021 to 2030 for each model were different (see Figure 10b). The GMDH-type ANN model's future prediction of TFR for India is better than others- it will come 1.41 children per women by 2030 (95% prediction interval: 1.44 - 1.38) which is less than the replacement level, or the average number of children per women required to sustain a steady population size of 2.1.

4. CONCLUSION

The study shows that the GMDH-type ANN model performs better than the ARIMA (1,2,2) model for predicting the TFR of India. The ARIMA model or other time series models might not be appropriate for long term predicting of TFR for India. Therefore, GMDH-type ANN model is particularly appropriate for analysing non-linear or unpredictable distribution data, such as fertility rates. GMDH-type ANN technique has more ability for predicting the TFR as compared to other time series models to inform the government to take policy actions in India.

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